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The Impacts of Tax Policy on Platform Markets in Digital Economy

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Abstract

This paper examines the effects of governmental policy in digital economy, especially the tax policy. We analyze how preferential tax policy affects the platform market and customer welfare. First, our findings demonstrate that prices have negative relation with preferential tax level in incompatible case and no relation in asymmetric compatible case. Corporation profits have positive relation with preferential tax level in asymmetric compatible case and no relation in incompatible case. Second, we derive a specific range in which preferential tax level could make it beneficial for both customers and platforms to improve compatibility. High compatibility is only good for platforms but not for customers when preferential tax level is too high. And oppositely, it's only good for customers when preferential tax level is too low, but platforms have no incentives to support for it.

Keywords: digital economy, preferential tax level, compatibility, two-sided platform

1. Introduction

With fast development of information technology, the world has stepped into the era of digital economy that features informatization and digital innovation (Adner R, Chen J & Zhu F., 2016). Despite the digital sector is less than 10% of most economies, China and the developed countries such as United States and Japan have taken the development of digital economy as a national strategy (Caillaud B & Jullien B., 2003). Under this background, it is very important to analyze the effects of governmental tax policy on digital economy, in order to provide useful reference for better development of digital economy.

As the most popular form in digital economy, platform has been adopted by more and more enterprises. Platform companies combine two groups of users with each other and that's why they are called two-sided platforms. Understanding of two-sided platforms starts from the study of credit card payment market. Rochet and Tirole (Bloch F & Demange G., 2018; Kind H J, Schjelderup G & Stähler F., 2013) find that a number of industries have adopted the same type of organization as the card payment industry, which relies heavily on platforms to connect two

groups of users like customers and businesses. Other kinds of two-sided platforms include software platforms combining users with developers (Mac OS, Windows, Android), transaction platforms combining buyers with sellers (Taobao, Amazon), media platforms combining readers with advertisers (YouTube, Time) and so on.

As the previous literature illustrate, theoretical models could be setup for two-sided platforms (Dou Y., 2014) from different industries, such as credit card market (Kotsogiannis C, Serfes K., 2010), newspapers (International Monetary Fund, 2018), video game platform (Bourreau M, Caillaud B & De Nijs R., 2018), software technology platform (Gaudin G, White A., 2014), e-book (Hao L & Fan M., 2014; Keen M & Lockwood B., 2010) and so forth.

This paper is relating to the e-book market including iPad and Kindle two two-sided platforms in competition (Hanna N., 2016). This is an asymmetric compatible market, which means that both iPad and Kindle provide e-book reading software iBooks and Kindle Reader respectively, but Kindle Reader is available on iPad and iBooks is not available on Kindle. More compatibility is what governments want to promote the platform market to have. Because governments need to do their best to prevent the trend of monopoly.

On the other hand, software related industries are more environmental and value-added. Hence comparing to hardware, governments generally make preferential tax policy for software related industries. Specifically, the major tax in China is value-added tax (VAT), which is included by over 130 countries' tax systems in diverse forms (Casadesus-Masanell R & Ghemawat P., 2006). VAT is a tax that is levied on every stage of the process for a product or service, while customers bear all the burden. In China, VAT has 13%, 9% and 6%, three levels for different categories of products or service. Hardware like iPad and Kindle should be levied 13% VAT on every stage. However, e-book only need to be levied 9% VAT. In addition, on publication stage, e-book firms could enjoy 50% VAT refund policy.

Our objective is to explore how tax impacts the platform market which are two-sided, especially their compatibility. And how do both the tax and compatibility influence customer welfare? How should governments make preferential tax policy concretely? For this purpose, we set up a Hotelling model between two competing platforms that earn profits from both hardware and software sides. Platforms could enjoy preferential tax policy in software side but not hardware side. And we perform two different cases, incompatible case and asymmetric compatible case.

Our findings demonstrate that prices have negative relation with preferential tax level in incompatible case and no relation in asymmetric compatible case. Corporation profits have positive relation with preferential tax level in asymmetric compatible case and no relation in incompatible case. Furthermore, in a specific range, preferential tax level could make it beneficial for both customers and platforms to improve compatibility. High compatibility is only good for platforms when preferential tax level is too high but not for customers. And oppositely, it's only good for customers when preferential tax level is too low, but platforms have no incentives to support for it.

The rest of this paper is organized as follows. Related literature is discussed in Section 2. We discuss the base model settings and notations in Section 3. We derive the equilibrium of two cases in Section 4 and compare them to obtain policy implications in Section 5. Section 6 concludes the paper. All proofs of propositions are relegated to the Appendix.

2. Literature Review

Many existing papers focus on the competition between two-sided platforms. Most of them are about symmetric platforms and few are about asymmetric platforms. Such as competition among intermediation service providers (Lin H, Guo H, Easley R F., 2017), open source and proprietary platforms (Rochet J-C & Tirole J., 2003; Gaudin G & White A., 2014), subscription-based and ad-sponsored platforms (Zhu F & Iansiti M., 2012) and licensing models (Kind H J, Koethenbuerger M & Schjelderup G., 2009).

Some papers are related with the compatibility. Maruyama and Zennyo (Kind H J, Koethenbuerger M & Schjelderup G., 2008), for example, find that compatibility depends on product life cycles. Casadesus-Masanell and Ruiz-Aliseda (Armstrong M., 2006) find out large

platforms' preference for incompatibility. Dou (Economides N & Katsamakas E., 2006) explains why platforms sell products through rival platform. Ron et al. (Hanna N., 2016) demonstrate that compatibility is related with profit foci.

There have been a few papers considering the issue of taxation in two-sided platforms with different focuses from ours. Kind et al. (Casadesus-Masanell R & Zhu F., 2010; Niculescu M F & Wu D J., 2014) compare the impacts of unit taxes and ad valorem on welfare and tax revenues in two-sided markets. Kind et al. (Maruyama M & Zennyo Y., 2013) discuss the role of tax on newspaper differentiation and investments in journalism. Kotsogiannis and Serfes (Casadesus-Masanell R & Ruiz-Aliseda F., 2008) consider the tax competition between countries. Another paper takes the effect of tax competition between countries into account (Rochet J, Tirole J., 2006). Bourreau et al. (Rochet J C, Tirole J., 2002) consider the tax effect on monopoly platform.

3. Model Settings and Assumptions

In this paper, we set up a competing model with the standard Hotelling model, including Apple's iPad and Amazon's Kindle as two platforms in competition. We assume that customers are uniformly distributed along a line from 0 to 1, and the two platforms situate at locations 0 and 1 on the line respectively. Each customer chooses to buy one of iPad and Kindle. Customer utility for a platform is the value that a customer could get from the platform except the price and mismatch cost from the platform and customer's preference. The mismatch cost is measured by the distance from platform to customer's location. We use platform 1 and 2 to index iPad and Kindle and denote the price as p_i, $i \in \{1,2\}$. Therefore, the utility for a customer at location x derived from platform i is denoted as u_i, and formulated as

$$u_i = v_i - cx - p_i$$

v_i denotes the value that a customer could derive from platform i by using its features like reading e-books and so on. Obviously, iPad provides much more special features than Kindle, such as music, game, video and so on. Then we assume that $v_1 > v_2$, and denote $\Delta v = v_1 - v_2$. And the mismatch cost c is the sum of mismatch cost from hardware c_h and mismatch cost from software c_s. Customers would choose to buy the one that offers more utility.

We assume that customers could read the same books from both platforms, and Apple and Amazon both can earn δ from a customer by selling e-books. Therefore, the profits π_i that platforms get from customers can be formulated as

$$\pi_i = p_i d_{ih}(1 - t_0) + \delta d_{is}(1 - t)$$

where d_{ih} denotes the number of customers who buy hardware from platform i, and d_{is} denotes the number of customers who buy e-books on platform i. Note that t_0 is the normal tax rate that platforms should pay for hardware sales, and t denotes the preferential tax rate that government offers in order to promote industrial development.

4. Equilibrium Analysis

In this section, we would analyze the equilibrium in different cases. Firstly, we analyze the incompatible case, in which each platform's software is not available on the other one. Then we analyze the asymmetric compatibility case, in which Amazon's Kindle Reader is available on Apple's iPad.

4.1 Incompatible Case

In this case, each platform's software service is not available on the other one. Consequently, the utilities u_1 and u_2 that a customer at location x could obtain from iPad and Kindle respectively are formulated as

$$u_1 = v_1 - cx - p_1,$$
$$u_2 = v_2 - c(1 - x) - p_2.$$

By letting $u_1 = u_2$, we can derive the indifferent customer's location $x* = \frac{\Delta v - (p_1 - p_2)}{2c} + \frac{1}{2}$.

Customers who are located from 0 to $x*$ have smaller mismatch cost than the indifferent customer from iPad, so they choose to purchase platform 1. The rest choose to purchase platform 2. Because of incompatible, customers who purchase hardware from a platform have to use the same platform's software, which means $d_{ih} = d_{is}$. Thus, we can formulate the profit functions of two platforms as follows:

$$\pi_1 = p_1 x*(1 - t_0) + \delta x*(1 - t),$$

$$\pi_2 = p_2(1 - x*)(1 - t_0) + \delta(1 - x*)(1 - t).$$

We denote $r_t = \frac{1-t}{1-t_0}$ as preferential tax level which indicates the relative level of preferential tax that software industry obtained from government. Then we can get the equilibrium results as summarized by the following proposition.

Proposition 1. When each platform's software is not available on the other one, the equilibrium prices are

$$p_1 = \frac{1}{3}(3c + \Delta v - 3\delta r_t),$$

$$p_2 = \frac{1}{3}(3c - \Delta v - 3\delta r_t),$$

the indifferent customer locates at $x* = \frac{1}{2} + \frac{\Delta v}{6c}$, and the equilibrium profits of two platforms are

$$\pi_1 = \frac{(1-t_0)(3c+\Delta v)^2}{18c},$$

$$\pi_2 = \frac{(1-t_0)(3c-\Delta v)^2}{18c}.$$

According to the equilibrium prices, when preferential tax level r_t increases, both p_1 and p_2 decrease and can even be negative if preferential tax level is high enough. Because preferential tax rate for software can make it more profitable than hardware. This will promote the platforms to subsidize customers' hardware purchases in order to make more profits from software side. However, equilibrium profits don't relate to preferential tax, although the prices are closely related to it. Government provides the same preferential taxation for both the two platforms in competition, such that neither of them can earn surplus income from the preferential policy. And both their profits decline with the rise of normal tax rate.

4.2 Asymmetric Compatible Case

In this case, we use notations with a tilde for outcome variables like $\tilde{p}$. When customers can use Kindle Reader on iPad, iPad buyers can choose any one of the two free software apps (Kindle Reader and iBooks) that costs less mismatch. In contrast, customers who purchase Kindle can't use iBooks. In consideration of the asymmetric compatibility case, we formulate the utility functions of two platforms as

$$\tilde{u}_1 = v_1 - c_h x - c_s \min\{x, 1 - x\} - \tilde{p}_1$$

$$\tilde{u}_2 = v_2 - c(1 - x) - \tilde{p}_2.$$

When the indifferent customer locates at $\tilde{x}* \geq \frac{1}{2}$, by letting $u_1 = u_2$ we have

$$v_1 - c_h\tilde{x}* - c_s(1 - \tilde{x}*) - \tilde{p}_1 = v_2 - c_h(1 - \tilde{x}*) - c_s(1 - \tilde{x}*) - \tilde{p}_2$$

Thus, we have $\tilde{x}* = \frac{\Delta v - (\tilde{p}_1 - \tilde{p}_2)}{2c_h} + \frac{1}{2}$. When $\tilde{x}* < \frac{1}{2}$, there are no customers who purchase iPad and use Kindle Reader at the same time, so the indifferent customer's location is the same as the one in the incompatible case, $\tilde{x}* = \frac{\Delta v - (p_1 - p_2)}{2c} + \frac{1}{2}$.

Hence, customers whose mismatch cost is lower than the indifferent customer chooses to buy platform 1 and the rest choose to buy platform 2. So, hardware sales of platform 1 and 2's are $\tilde{x}*$ and $(1 - \tilde{x}*)$ respectively. Software sales is related to $\tilde{x}*$. When $\tilde{x}* < \frac{1}{2}$, customers who buy iPad will choose to use software iBooks. When $\tilde{x}* \geq \frac{1}{2}$, customers belonging to $(\frac{1}{2}, \tilde{x}*)$ will buy iPad but use software Kindle Reader. Then we can formulate profit functions in this case as

$$\tilde{\pi}_1 = \tilde{p}_1\tilde{x}*(1 - t_0) + \delta \min\{\frac{1}{2}, \tilde{x}*\}(1 - t)$$

$$\tilde{\pi}_2 = \tilde{p}_2(1 - \tilde{x}*)(1 - t_0) + \delta \max\{\frac{1}{2}, 1 - x*\}(1 - t)$$

We can derive the equilibrium results as showed in the following proposition.

Proposition 2. When Amazon's software Kindle Reader is available on Apple's hardware, the equilibrium prices are

$$\tilde{p}_1 = \frac{1}{3}(3c_h + \Delta v),$$

$$\tilde{p}_2 = \frac{1}{3}(3c_h - \Delta v),$$

the indifferent customer locates at $\tilde{x}* = \frac{1}{2} + \frac{\Delta v}{6c_h}$, and the equilibrium profits of two platforms are

$$\tilde{\pi}_1 = (1 - t_0)\left(\frac{(3c_h + \Delta v)^2}{18c_h} + \frac{\delta}{2}r_t\right),$$

$$\tilde{\pi}_2 = (1 - t_0)\left(\frac{(3c_h - \Delta v)^2}{18c_h} + \frac{\delta}{2}r_t\right).$$

In this case, according to proposition 2, we can see that equilibrium prices are not related to preferential taxation. This is because even though $\tilde{x}*$ is still lager than 0.5, iPad purchasers have option to choose which software they would use. There is no longer incentive for platforms to subsidize customers' hardware purchases. In addition, profits of both platforms will increase with the increasing preferential tax level r_t. The equilibrium prices and product sales are independent of r_t, so the higher r_t improves profits from income of software sales.

5. Comparison of the Two Cases

Consequently, we compare the equilibrium results in two cases and examine how the preferential tax level r_t influences the pricing and compatibility strategy of both platforms.

By comparing the equilibrium prices in two cases as showed in proposition 1 and 2, we can get the following proposition.

Proposition 3. If and only if $r_t > \frac{c_s}{\delta}$, both platforms would like to charge lower price in incompatible case than in asymmetric compatible case, which means $\tilde{p}_i > p_i$.

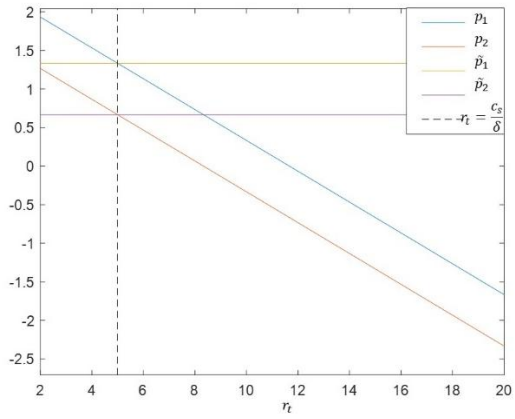

Figure 1. Comparison of price

Without loss of generality, we denote $c = 2,\ c_h = 1,\ \Delta v = 1,\ \delta = 0.2$, then $r_t = \frac{c_s}{\delta} = 5$ and we have figure 1. Just like what has been demonstrated in proposition 1, higher preferential tax level r_t can lead to lower price p_i. So, when r_t becomes high enough, platforms will charge lower price in the incompatible case.

Proposition 4. If and only if $9(\delta r_t - c_s) + \Delta v^2(\frac{1}{c_h} - \frac{1}{c}) > 0$, both platforms would like to support for the asymmetric compatibility. And the incentives to provide Kindle Reader on iPad increase as the preferential tax level r_t increases.

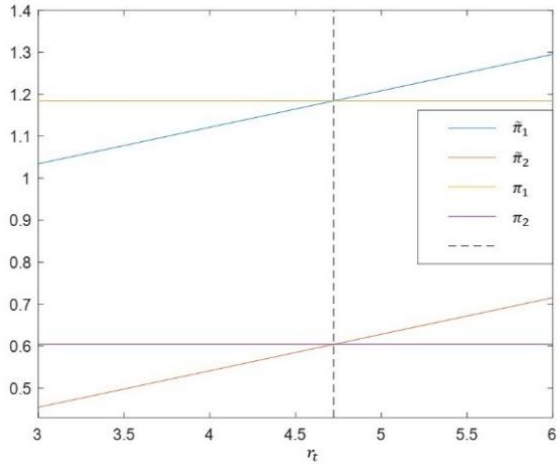

Figure 2. Comparison of profits

Here we denote

$c = 2,\ c_h = 1,\ \Delta v = 1,\ \delta = 0.2,\ t_0 = 0.13$, then $r_t = 4.72$ and we have figure 2. The preferential taxation policy is a method that usually used to promote the development of software industry in many countries. As has been demonstrated by Ron et al. (2016), profit of Apple focuses on hardware sales and profit of Amazon focuses on software. However, we find out that when preferential tax level r_t is higher, Apple is fonder of making Kindle Reader available on iPad, which is counterintuitive. Despite in the asymmetric compatible case, higher preferential tax level r_t makes Amazon more competitive, Apple choose to support for compatible case. This is because higher r_t makes Apple no profit on hardware sales, and it has no advantage on software competition. Apple would rather choose to support for the compatible case so that it can keep its competitive advantage on hardware sales.

Proposition 5. If and only if

$$18c_s + 9c - 36\delta r_t + \Delta v^2\left(\frac{1}{c_h} - \frac{1}{c}\right) > 0 \quad ,$$

compatible case generates greater customer welfare than the incompatible case. And the customer welfare advantage decreases with the increasing preferential tax level r_t.

This is because r_t has positive correlation with customer welfare in incompatible and no correlation in asymmetric compatible case, which is showed in the proof of proposition 5. Hence, higher r_t is good for promoting platforms more compatible, but not for customer welfare.

By comparing the results in proposition 4 and 5, we can obtain the important results illustrated in the following proposition.

Proposition 6. As the preferential tax level r_t increases, platforms' incentives to support for compatible case become higher, while customers' incentives become lower. Specifically,

(1) when $r_t < \frac{1}{\delta}\left[c_s - \frac{\Delta v^2}{9}\left(\frac{1}{c_h} - \frac{1}{c}\right)\right]$, asymmetric compatible case is better for customers, but not for platforms;

(2) When

$$\frac{1}{\delta}\left[c_s - \frac{\Delta v^2}{9}\left(\frac{1}{c_h} - \frac{1}{c}\right)\right] < r_t < \frac{1}{\delta}\left[\frac{1}{2}c_s + \frac{1}{4}c + \frac{\Delta v^2}{36}\left(\frac{1}{c_h} - \frac{1}{c}\right)\right]$$

asymmetric compatible case is better for both customers and platforms;

(3) When $r_t > \frac{1}{\delta}\left[\frac{1}{2}c_s + \frac{1}{4}c + \frac{\Delta v^2}{36}\left(\frac{1}{c_h} - \frac{1}{c}\right)\right]$, asymmetric compatible case is better for platforms, but not for customers.

6. Conclusion

Motivated by the phenomenon that comparing to hardware, many countries provide preferential taxation for software related industries, we study the impacts of China's preferential tax policy on the compatibility on two-sided platforms. We derive some counterintuitive results that are worth to be highlighted. First, in incompatible case, preferential tax level is negatively related with prices, but has no relation with platform profits. In asymmetric compatible case, preferential tax level is positively related with platform profits, but has no relation with prices. Second, as the preferential tax level increases, incentives of platforms to provide more compatibility increase, but customer welfare would be decreased. Therefore, we derive a specific range in which preferential tax level could make it beneficial for both customers and platforms to improve compatibility. High compatibility is only good for platforms when preferential tax level is too high but not for customers. And oppositely it's only good for customers when preferential tax level is too low, but platforms have no incentives to support for it.

In this paper, we model the platforms that earn profits from both hardware and software sides. However, there are many apps that could be used for free. It would be interesting to study platforms that earn only from hardware side or those have opposite profit patterns in the future.

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Appendix A

Proof of proposition 1.

As $x* = \frac{\Delta v-(p_i-p_m)}{2t} + \frac{1}{2}$ functions (5) (6) could be rewritten as follows

$$\pi_1 = [p_1(1-t_0) + \delta(1-t)]\left(\frac{\Delta v-(p_1-p_2)}{2c} + \frac{1}{2}\right)$$

$$\pi_2 = [p_2(1-t_0) + \delta(1-t)]\left(\frac{1}{2} - \frac{\Delta v-(p_1-p_2)}{2c}\right)$$

By solving the first order conditions, $\frac{d\pi_1}{dp_1} = 0$ and $\frac{d\pi_2}{dp_2} = 0$, we have

$$p_1 = \frac{1}{2}(p_2 + c + \Delta v - \delta\frac{1-t}{1-t_0})$$

$$p_2 = \frac{1}{2}(p_1 + c - \Delta v - \delta\frac{1-t}{1-t_0}).$$

Solve the equations above and derive the equilibrium prices in proposition 1. And then the other equilibrium results could be derived. Proof of proposition 1 is completed.

Proof of proposition 2.

When $\tilde{x}* \geq \frac{1}{2}$, we have $\tilde{x}* = \frac{\Delta v-(\tilde{p}_i-\tilde{p}_m)}{2t_h} + \frac{1}{2}$.

Then the profit functions can be rewritten as

$$\tilde{\pi}_1 = \tilde{p}_1\frac{\Delta v-(\tilde{p}_1-\tilde{p}_2)+c_h}{2c_h}(1-t_0) + \frac{1}{2}\delta(1-t)$$

$$\tilde{\pi}_2 = \tilde{p}_2(1 - \frac{\Delta v-(\tilde{p}_1-\tilde{p}_2)+c_h}{2c_h})(1-t_0) + \frac{1}{2}\delta(1-t)$$

By solving the first order conditions $\frac{d\tilde{\pi}_1}{d\tilde{p}_1} = 0$ and $\frac{d\tilde{\pi}_2}{d\tilde{p}_2} = 0$, we have

$$\tilde{p}_1 = \frac{1}{2}(\tilde{p}_2 + c_h + \Delta v)$$

$$\tilde{p}_2 = \frac{1}{2}(\tilde{p}_1 + c_h - \Delta v).$$

Solve the equations above and derive the equilibrium prices in proposition 2. And then the other equilibrium results could be derived.

When $\tilde{x}* < \frac{1}{2}$, we could obtain the same results as proposition 1. And $\tilde{x}* = \frac{1}{2} + \frac{\Delta v}{6t} > \frac{1}{2}$. This is impossible.

Proof of proposition 2 is completed.

Proof of proposition 3.

According to proposition 1 and 2, we can derive

$$\tilde{p}_1 - p_1 = \frac{1}{3}(3c_h + \Delta v) - \frac{1}{3}(3c + \Delta v - 3\delta r_t) = \delta r_t - c_s$$

If and only if $r_t > \frac{c_s}{\delta}$, the equation above would be bigger than 0.

In the same way, we have

$$\tilde{p}_2 - p_2 = \frac{1}{3}(3c_h - \Delta v) - \frac{1}{3}(3c - \Delta v - 3\delta r_t) = \delta r_t - c_s$$

If and only if $r_t > \frac{c_s}{\delta}$, the equation above would be bigger than 0.

Proof of proposition 3 is completed.

Proof of proposition 4.

According to proposition 1 and 2, we can derive

$$\tilde{\pi}_1 - \pi_1 = \tilde{\pi}_2 - \pi_2 = \frac{1}{18}\frac{(1\text{-}t_0)}{18}[9(\delta r_t - c_s) \cdot \Delta v^2(\frac{1}{c_h} - \frac{1}{c})]$$

If and only if $9(\delta r_t - c_s) + \Delta v^2(\frac{1}{c_h} - \frac{1}{c}) > 0$, the equation above would be bigger than 0.

Proof of proposition 4 is completed.

Proof of proposition 5.

Total customer welfare in incompatible case can be formulated as

$$U(x) = \int_0^{x*}(v_1 - cx - p_1)dx + \int_{x*}^1[v_2 - c(1-x) - p_2]dx$$

whereas $x* = \frac{1}{2} + \frac{\Delta v}{6c}$.

Total customer welfare in compatible case can be formulated as

$$\tilde{U}(\tilde{x}) = \int_0^{\tilde{x}*}(v_1 - c_h\tilde{x} - c_s\, min\{\tilde{x}, 1-\tilde{x}\} - p_1)d\tilde{x} + \int_{\tilde{x}*}^1[v_2 - c(1-\tilde{x}) - p_2]d\tilde{x}$$

whereas $\tilde{x}* = \frac{1}{2} + \frac{\Delta v}{6c_h}$.

Hence, by solving $\tilde{U}(\tilde{x}) - U(x) > 0$, we can obtain $18c_s + 9c - 36\delta r_t + \Delta v^2(\frac{1}{c_h} - \frac{1}{c}) > 0$.

Proof of proposition 5 is completed.

Proof of proposition 6.

According to proposition 4 and 5, we have $r_t > \frac{1}{\delta}[c_s - \frac{\Delta v^2}{9}(\frac{1}{c_h} - \frac{1}{c})]$ and $r_t < \frac{1}{\delta}[\frac{1}{2}c_s + \frac{1}{4}c + \frac{\Delta v^2}{36}(\frac{1}{c_h} - \frac{1}{c})]$, respectively.

By comparing the two equations, we can obtain the results in proposition 6.