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Wavelet Analysis of the Population Growth in a Depopulation Context Within Two Balkans Countries

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Abstract

Serbia and Bulgaria have been selected in order to demonstrate how the principles of wavelet engines are applied to real data for the population growth rate (POPG). The annual time series data are retrieved from the UN database covering the period from 1990 to 2021. The major part of relevant wavelet coefficients (those coefficients exceeding threshold bounds) are concentrated in the first few scales. It was observed that the more significant ''wave'' behavior over wavelet coefficients and scales for both countries was revealed with MODWT transformation in contrast to the discrete wavelet transform (DWT). The main use of the Multi-Resolution Analysis (MRA) was the separation of the ''signal'' of the original POPG series from its noise, at a specific decomposition level. In both countries persistent behavior in the POPG series was found. A proximate examination of the wavelet threshold noise series reveals moderate differences between the countries. The large outlying observation in 2020 is accurately identified for the POPG series of both countries. There is no doubt that the theoretical background presented in this study might be relevant for each of the countries within this study observing in one or the other way.

Keywords: Wavelet analysis, DWT, MODWT, MRA, population growth, Serbia, Bulgaria

1. Introduction

The ageing of the 'baby boom' generation and the continuation in declines of fertility and old-age mortality are changing the population balance in developed (Bloom & Canning, 2006) and less developed countries from young to old. The changes in vital rates generate considerable changes in the population size, the rate of population growth, and the age structure of population (Lee, 2011). The explanation for the broad widespread presence of below replacement fertility and the extent of negative natural increase could be understood with the predominant concept introduced by Keyfitz in 1971 - population momentum (Schoen, 2018). Thus, the well-known concept of *growth momentum* refers to the influence of a changing age structure of a population on its future growth rate. Growth momentum refers to periods of fertility changes, like the historical transition from high to low fertility as well as to transitions from sub-replacement level fertility to replacement level (Lesthaeghe & Surkyn, 2021). Accordingly, the population growth will not stop after the TFR will reach 2 children or when a replacement level will take place. Even if there is a high fertility, which is the main cause for population growth, humankind should adjust to the long-run replacement level of 2.1 children per woman in order not to continue to experience demographic changes (Bloom & Canning, 2006). The growth rate remains negative if the sub-replacement fertility level with two children lasts for a few decades, and when the ageing mechanism has started to take place (Lesthaeghe & Surkyn, 2021). As a result of the continued effect of old age structure; the death rate exceeds the birth rate. In general, the decline in population growth can be attributed to a very great degree to the decreasing surplus of births over deaths (Avdeev et al., 2011). Furthermore, in the opinion of Lesthaeghe and Surkyn (2021), the combination of negative growth momentum and emigration would be a jointly strengthening mechanism leading to a more rapid population falling down, because it is well known that emigration would further reduce the younger age groups.

Serbia and Bulgaria have been selected for this research study because these countries are specific in terms of population growth over the past three decades. This whole period is characterized by a pronounced negative population growth in both countries; the situation for Bulgaria is especially alarming. The population of Serbia is characterized by a long-term tendency to slow down population growth. Today, the population of Serbia is facing a fertility rate below the level needed for simple population renewal. According to the population projections of the Serbian Statistical Office (2014), in the variant of constant fertility, the population of the Republic of Serbia will decrease in 2041 by 14% (Djurdjev & Arsenović in RZS, 2015). Even with the variant of high fertility, the population size in Serbia will decrease in the next three decades. Low and negative rates of natural increase, as well as high average age of the population cause intensive depopulation in all parts of the country (ibid). Therefore, according to these authors, depopulation processes can be characterized as basic demographic, but also economic and social problems of the population of Serbia, and demographic growth as a limiting development factor. Bulgaria is the country with the greatest projected population decline in Europe, ending in 2060 with a population size around one third less than in 2010 (Lanzieri, 2013). In addition, Lanzieri (2010) also mentions that it is projected Bulgaria to have a relatively negative population momentum until 2060, combined with the highest multiplier for mortality. Thus, this present paper proposes that a population growth function is dependent on time. Therefore, population growth has been assumed to change with respect to time. Hence, the main contribution of this research work is the explicit introduction of wavelet-based approach into the population growth model. An interesting research question, addressed within this research work, is whether population growth (or decline) in these two countries persists for a long time and which forces have contributed most to its overall variation. The remainder of this research work is organized as follows. Section 2 investigates the relevant theoretical background of population growth. Section 3 introduces data used and wavelet methodology approach. Section 4 deals with main findings and discussion. The last section gives some concluding remarks.

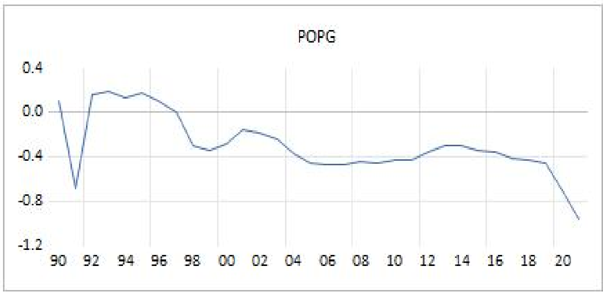
2. Theoretical Background

Population growth results as a function of birth, death, and immigration and emigration rates and is caused by both endogenous and exogenous forces (Mohamed & Schachler, 2016). The endogenous impacts mostly affect birth rates and death rates, which are affected through the age groups and per capita income of the given population. The employment and labor payment are those endogenous forces that mostly affect immigration and emigration. It is believed that the exogenous impacts mostly affect immigration and emigration. Exogenous variables that may affect population growth within the concerned population might be medical factors, social security models, cultural and local development, as well as tax and stimulants or sanctions for child-bearing. Thus, according to the underlying *exogenous growth theory*, changes in population growth rates do not have a long run impact on the economic growth of a country but only have an effect on the transitional dynamics, while the primary *endogenous growth theory* claims that changes in population growth rates determine both the transitional dynamics and the long-lasting economic growth of the country (Mohamed & Schachler, 2016). Prskawetz and Feichtinge (1995) suggest that there is a dependence of the population growth rate on the development phase and in turn that development is affected by past population growth rates. Thus, these authors assumed a positive relationship between population growth and the standard of living presented by the per capita product. Furthermore, Prskawetz and Feichtinge (1995) point out that when there is lack of technological change, the population growth represents a vital stimulus for the whole economy. Models of *endogenous technological change* based on increasing returns in most cases manifest a positive relationship between size of the population and the rate of growth of income per capita (Peretto, 1998). Thus, the theory based on these models predicts that large countries grow faster than small ones and those countries with growing populations show an increasing growth rate.

Read and LeBlanc (2003) developed the *multi-trajectory model*. This model covers the complex relationships between population growth and societal change. There is a suggestion by these scholars that understanding of the different trajectories that different societies and cultures have followed is a kind of dynamic and that also the changes in population growth and density as well as in society have been occurring continuously over a period of time. In their theoretical approach, Robinson and Srinivasan (1997) emphasize that the evolution of population, technology and utilization of resources as well as government policymaking and social institutions are the jointly endogenous outcome of the decisions of agents in an economy. All theories including the *Neo-Malthusian perspectives* together suggest a few simple views regarding population growth factors: understanding and specific knowledge of local circumstances, processes, and significant political, economic and institutional factors (Bremner, Lopez-Carr & Davis, 2010). Robinson and Srinivasan (1997) link up with the classic "Malthusian debate", regarding the nature of the relationship between technical progress and population change, but the main concerns are the matters raised by "neo-Malthusians", i.e., the frailty of the environment, the influence of economic and population growth on the capital stock existing in nature and with the whole process of economic, political and social change which these authors call development. Marsiglio (2010) provides the typical answer on the question of the optimal population size. In order to determine the optimal population size, he proposes those populations which under given circumstances ensures the largest social welfare. Furthermore, when Marsiglio (2010) defines a *social welfare function*, he considers two major criteria: *average utilitarianism* and *total utilitarianism*. In total utilitarianism, the goal is to maximize total wellbeing in the economy, and that is the average welfare multiplied by the population size. Implicit for this criterion is that the utility of not being born is zero. According to this criterion increasing the size of population always improves welfare if his/her utility is greater than a critical level, which is zero in total utilitarianism. Consequently, it implies that population should be increased for an unspecified period of time, even if average utility may approach zero. Instead, average utilitarianism proposes that the population should be as small as possible, because this maximizes average welfare (Marsiglio, 2010; Palivos & Yip, 1993).

3. Data and Methods

Within our research study it is demonstrated how the principles of wavelet engines are applied to real data for the population growth rate. Furthermore, in this research paper, the application of wavelets is presented having some analogy with demographic time series and in particular population dynamics and growth, in which scope this research is drafted. In order to analyze and understand the changing pattern of population growth in these two countries, the period of the last three decades has been considered. Thus, to show the population dynamics outlined earlier in this paper, the Serbian and Bulgarian population growth rate data are considered. The data are retrieved from the database of the UN (2022), (https://population.un.org/wpp). These are annual time series data covering the period from 1990 to 2021. Thus, the series of our research interest is the population growth rate (POPG) in Serbia and Bulgaria. A sample size of 32 observations for each of the countries provides a formal assurance for decent results. Since the number of available observations is 32 a dyadic adjustment using the series mean was employed to achieve dyadic length (Bilen & Huzurbazar, 2002). Figures 1-2 show POPG series for Serbia and Bulgaria, respectively.



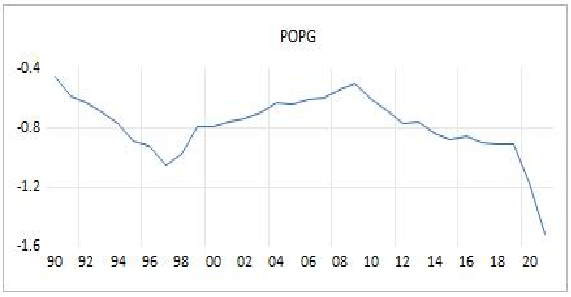


Figure 1. Population growth rate in Serbia: 1990-2021 (left)

Figure 2. Population growth rate in Bulgaria: 1990-2021 (right)

Source: Author's design based on UN data Source: Author's design based on UN data

Wavelet theory was born in the middle of 1980s. Through wavelet analysis, the data set goes through a preliminary processing through de-noising and is then segmented in numerous time scales (Singh & Loh, 2010). The fields of application of wavelet analysis have grown rapidly in the last decades. Since the 1990s, the wavelet analysis expanded with high-speed and its usage has covered large areas in physics, geophysics, astronomy, epidemiology, signal processing and so on (Singh & Loh, 2010). Therefore, the wavelet application includes four attractive and most often used empirical fields of wavelet analysis: transforms, variance decomposition, thresholding and outlier detection (Bilen & Huzurbazar, 2002; HIS, 2020). The wavelet transformation decomposes a time series in a number of coefficients connecting to a scale or horizon, where the parts of each number of coefficients are connected with a certain location (Conlon, Cotter & Gençay, 2018). The Wavelet decomposition is able to decompose a time series in its long-run behavior (smooths) and short run behavior (details), (IHS 2020). Wavelet analysis also has the ability to reveal signal aspects that are not possible with other analysis techniques, e.g., trends, breakdown points, discontinuities (Merry & Steinbuch, 2005). Wavelet analysis performs a local time scale decomposition of the signal, or an estimation of its spectral features as a function of time or space (Cazelles et al., 2014). As mentioned by Velarde, Meredith and Weyde (2016) there are two major patterns of the wavelet transform: the continuous wavelets transform (CWT) and the discrete wavelets transform (DWT). Accordingly, the CWT is mainly used for pattern analysis or feature identification in signal analysis. Using the CWT any changes in the direction of the series over time can be identified. The DWT is applied for compression and reconstruction. A wavelet transform of a discrete time series (DWT) can be usually expressed as a simultaneous representation of the time series with both time and frequency functions (Klarl, 2016). In the similar way, when the variance decomposition was performed then the Maximum Overlap Discrete Wavelet Transform (MODWT) and Daubechies filter are used (Bilen & Huzurbazar, 2002). Relative to the Haar wavelet, when selected for the subsequent analysis, the Daubechies has smaller changes and less distortion in the location of features (Conlon, Cotter & Gençay, 2018). The MODWT is a mathematical technique that projects a time series on a series of orthogonal basic functions, i.e., wavelets, occurring in a set of wavelet coefficients or filtered time series related to a scale or horizon (Conlon, Cotter & Gençay, 2018). MODWT is a non-decimated form of wavelet transformation about which can be seen to exactly decompose the overall variance and covariance of a time series into sums related to a certain scale (Conlon, Cotter & Gençay, 2018). Therefore, in order to prevail over the limitations associated with the DWT, the maximum overlap discrete wavelet transform (MODWT) is often used. There are several advantages of MODWT over the DWT (Conlon & Cotter, 2011). Thus, the MODWT allows an alignment of wavelet scaling and coefficients with the original data. The MODWT could also handle any sample size N, whereas the DWT confines the sample size to a multiple of 2^j. Furthermore, with MODWT the wavelet variance and covariance effectively at distinct scales can be calculated. CWT uses a small wave or wavelet[1] $\varphi(t)$ which represents the function of a time parameter t. The wavelet coefficients, W (τ, ϵ), are associated with a time series $f(t)$ and they usually can be calculated as:

$$\mathrm{W}\ (\tau, \epsilon) = \sum_{t=1}^{N} f(t)\varphi^{*}\left[\frac{t-\tau}{\epsilon}\right], \tag{1}$$

where φ^{*} denotes the complex conjugate of the wavelet, $\epsilon > 0$ is the scale connected with the transformation, $[-\alpha < \tau < \alpha]$ is the position in frequency domain with fixed location and $\frac{1}{\epsilon}$ is a normalization factor (Conlon, Corbet & McGee, 2021). In other words, as put by Klarl (2016), τ controls the location of the wavelet, thus if τ > 1, the wavelet is stretched and when τ < 1, then the wavelet is compressed. By discretizing the parameters, *a* and *b*, a discrete form of the wavelet transform (DWT) is obtained (eq.2-3). The approach becomes more effective if dyadic values of *a* and *b* parameters have been used[2].

$$a=2^{j} \qquad b=2^{j}k \qquad j,k \in Z \tag{2}$$

where Z is a collection of positive whole numbers. The appropriate discretized wavelets $\varphi_{j,k}$ defined as:

$$\varphi_{j,k}(t)= 2^{-\frac{j}{2}}\ \varphi\,(2^{-j}\,t-k) \tag{3}$$

$\varphi_{j,k}$ forms an both orthogonal and normalized base. As wavelet basic functions are in practice location and scale transformations of a single function, then this is a most suitable tool for the Multi Resolution Analysis (MRA) which analyses a signal at different frequencies with varying resolutions. Indeed, the general idea in wavelet theory is described as Multi Resolution Analysis (MRA) which makes reference to the approximation of a series at each scale (and up to all scales) λ_j.

$$\text{Thus, for each } m = 1, \ldots, M,\ y = w'W = \sum_{j=1}^{m} D_j + S_m \tag{4}$$

where the $D_j = w'W$ and $S_m = v_m' V_m$ are *T*-dimensional vectors of the *j*-th detail and the *m*-th smooth series (IHS, 2022). In addition, the detail and smooth series are associated with changes and average at scale λ_j, respectively, in the series y.

In the discrete wavelet analysis, a signal can be shown by its approximatively values and details (Pnevmatikos, 2010). In order to analyze the high frequencies, the signal is passed through a series of high-pass filters, which show connection with the details. In order to analyze the low frequencies, the signal is passed through a series of low-pass filters, which are related to the approximations. The DWT may be formed as an orthonormal transformation. The discrete transform is no longer orthonormal when there are available the full set of observations with $k = 1, \ldots, T$. This structure is usually referred to the maximal overlap discrete wavelet transform (MODWT) or non-decimated DWT (IHS, 2022). In addition, following Porwik and Lisowska (2004), the Haar wavelet and scaling functions can be defined by the following formulas as in eq. (5-6):

$$\varphi(t) = \begin{cases} 1, & \text{for } t \in [0, \frac{1}{2}), \\ -1, & \text{for } t \in [\frac{1}{2}, 1), \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

Similarly to the wavelet function, the Haar scaling function can be defined as in eq. (6):

$$\Phi(t) = \begin{cases} 1, \text{for } t \in [0,1), \\ 0, \text{for } t \notin [0,1). \end{cases} \tag{6}$$

$$W_{j,t} = \sum_{m=0}^{L_{j-1}} h_{j,m} X_{t-m \bmod N'} \ \text{ for } t = 0, \ldots. N-1 \tag{7}$$

$$V_{j,t} = \sum_{m=0}^{L_{j-1}} g_{j,m} X_{t-m \bmod N'} \ \text{ for } t = 0, \ldots. N-1, \tag{8}$$

The term mod indicates *operations modulo*. The process $\{W_{j,t}\}$ is designated as the *j*-th level MODWT wavelet coefficients and the process $\{V_{j,t}\}$ is designated as the *j*-th level MODWT scaling coefficients (Bašta, 2011). Since the *j*-th level wavelet coefficients $\{W_{j,t}\}$ are obtained with the help of the linear filter $\{h_{j,t}\}$, which is approximately an ideal filter with the nominal frequency band [2 − (j + 1), 2 − j], it is instinctively easy to understand that the *j*-th level wavelet coefficients are connected with the same frequency range and capture the dynamics of the stochastic process $\{X_t\}$ in this frequency span. Similarly, *j*-th level scaling coefficients $\{V_{j,t}\}$ are obtained with the help of the linear filter $\{g_{j,t}\}$, which is approximately an ideal low-pass filter for the frequency range [0, 2 −(j+1)], (Bašta, 2011). Thus, it is instinctively understandable that the *j*-th level scaling coefficients are linked with the same frequency range to take into account long-term dynamics of the stochastic process $\{X_t\}$ because it is known that low frequencies concur with high periods and thus trending movements.

4. Research findings and Discussion

From the unit root test results for both countries presented in Table 1, it was clear that the *p*-value associated with the ADF unit root test is 0.8799 and 0.9746, respectively, and it indicates that they are too high to reject the null hypothesis of a unit root at any statistically significance level. Hence, these findings demonstrate how wavelets can be used as an exploratory tool for non-stationary time series. However, based on further results of our unit root test it can be concluded that POPG series for both countries have persistence, i.e., it was found to have a unit root and therefore it is not an automatically non-stationary series.

Table 1. Serbian and Bulgarian POPG: Unit Root Test

Null Hypothesis: POPG has a unit root				
Exogenous: Constant				
Lag Length: 0 (Automatic based on SIC, max lag=7)	Serbia		Bulgaria	
	t-Statistic	Prob.	t-Statistic	Prob.
Augmented Dicky-Fuller test statistic	-0.4894	0.8799	0.3044	0.9746
Test critical values: 1%level	-3.6702		-3.6702	
5%level	-2.9640		-2.9640	
10%level	-2.6210		-2.6210	

Source: Author’s calculation

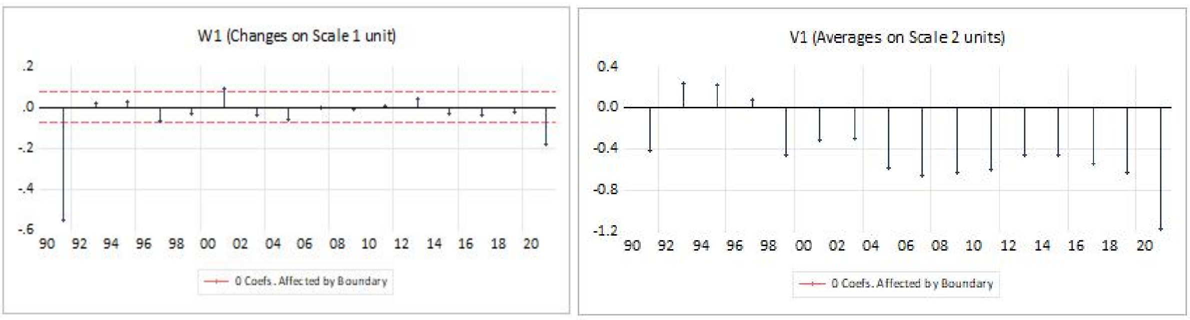


Figure 3. Serbian POPG: Discrete Wavelet Transform

Source: Author’s design based on UN data

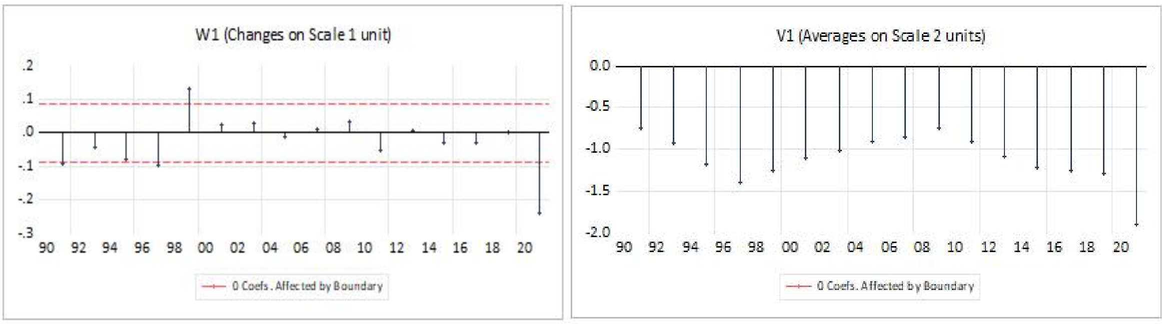


Figure 4. Bulgarian POPG: Discrete Wavelet Transform

Source: Author's design based on UN data

The output in Figure 1-4 is a spool object with the spool tree showing the original series (Figures 1-2), as well as wavelet and scaling coefficients (Figures 3-4). From wavelet transform results it can be seen clearly that at the first scale, the wavelet decomposition successfully breaks the frequency spectrum into two equal portions: the low and high frequency parts, respectively for both countries. The low frequency part is associated with the scaling coefficients VV in contrast to the high frequency part which is associated with the wavelet coefficients WW (Bilen & Huzurbazar, 2002; IHS, 2022). It is evident also that the spectra characterizing the wavelet coefficients are significantly less pronounced than those characterizing the scaling coefficients. This is a further indication that both Serbian and Bulgarian population growth series are possibly non-stationary. From Figure 3-4 it can be noticed that the wavelet plot has two dotted red lines. These lines show the ±1±1 standard deviation of the coefficients on that scale. It is especially interesting in illustrating which wavelet coefficients should be shrunk to zero, i.e., which coefficients are insignificant in wavelet shrinkage applications. Thus, the coefficients exceeding some threshold bound (the standard deviation in our case) need to be retained, while the remaining coefficients are shrunk to zero. From this point of view, it can be seen that the larger number of wavelet coefficients at scale 1 for both countries may be discarded as not useful. This is some other evidence that high frequency forces in both Serbia and Bulgaria series are not very pronounced.

Furthermore, in order to identify which scales or frequencies have influence on the POPG series behavior, this analysis will not be limited only to the first scale but will be extended using the maximum overlap discrete wavelet transform (MODWT) with the Daubechies filter of length 6 (Figures 5-6). It can be observed the significantly more pronounced ''wave'' behavior over wavelet coefficients and scales for both countries. This is due to the fact that the MODWT is not an orthonormal transform and since it uses all of the available observations (Bilen & Huzurbazar, 2002). Put differently, models retain their momentum while they transform gradually. On the scale 1 of the MODWT for both countries (Figure 5-6), it is seen that there are only a few wavelet coefficients that show significant spikes or exceed the threshold bounds. At scale 2, it is obvious that the transient features are permanent, but after that there is no contribution. Thus, at scale 2, Figure 5 for Serbia, there are only a few coefficients which exceed the threshold bounds. Even though these coefficients are in the lower half of the frequency scope, these coefficients are not in the instant proximity of frequency 0. On the other hand, the scaling coefficients at the final scale (scale 5) are approximately slightly smaller for the Serbia POPG series (0.30) and twice as large (0.80) for the Bulgaria POPG series from the largest wavelet spectrum (0.40) and (0.40) for Serbia and Bulgaria, respectively, which manifested themselves on scales 1 and 2. All these findings are indications that lower frequency forces dominate those at higher frequencies. Boundary coefficients are the result of longer filters and higher scales. Clearly, as the scale is increased, boundary coefficients ingest the whole set of coefficients. The number of boundary coefficients is greater for Bulgaria POPG series than for Serbia POPG series.

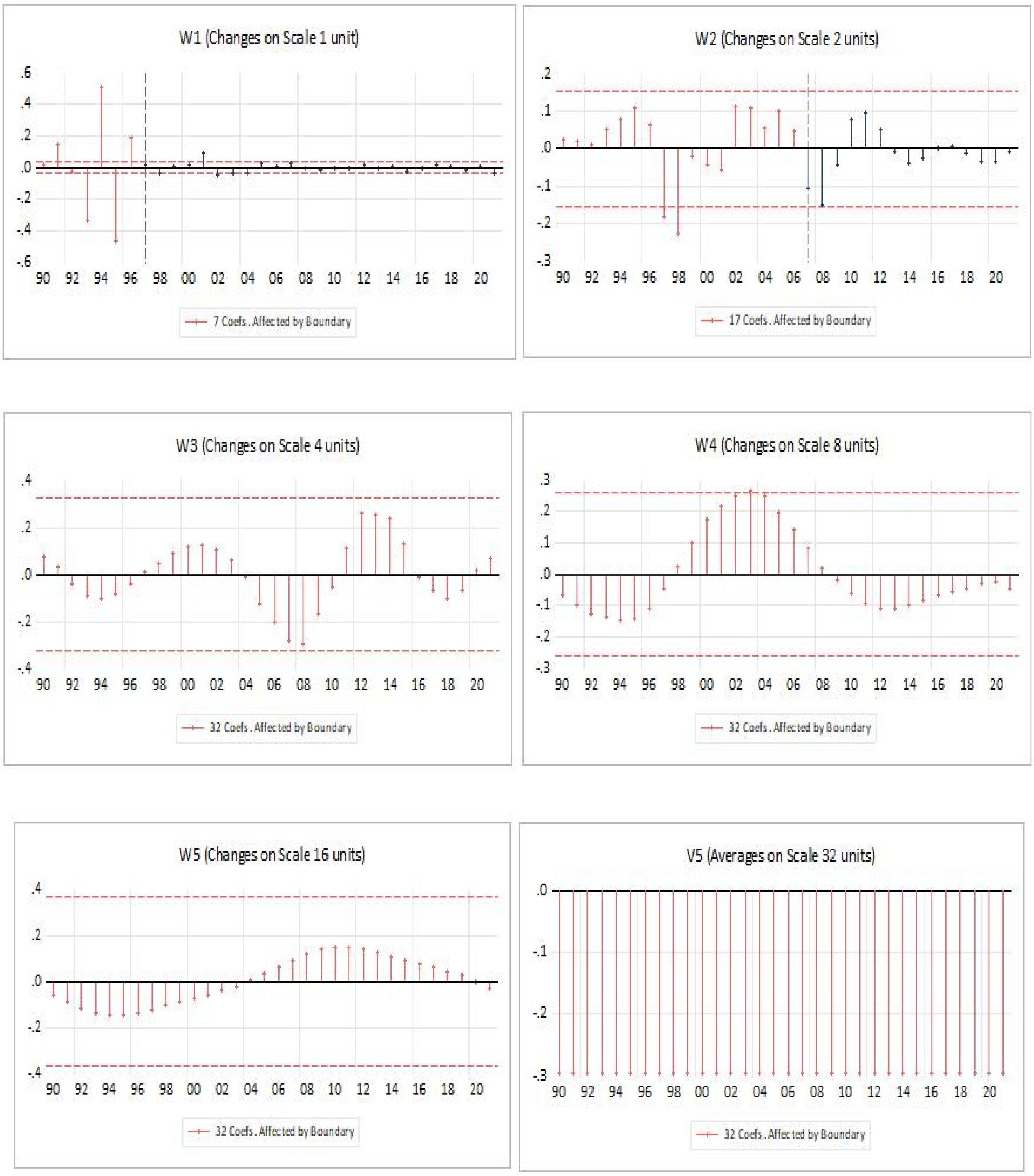


Figure 5. Serbian POPG: MODWT

Source: Author's design based on UN data

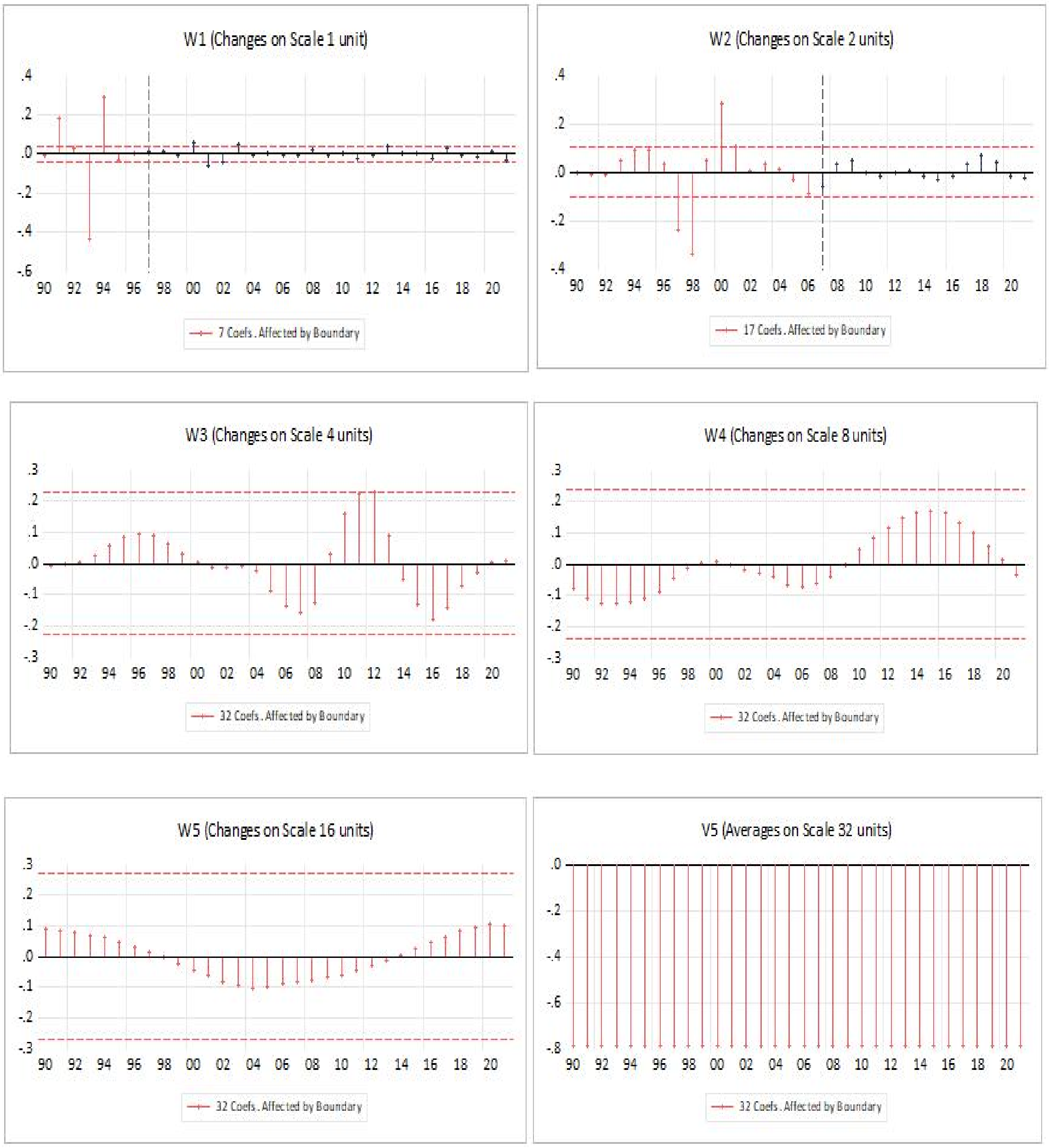
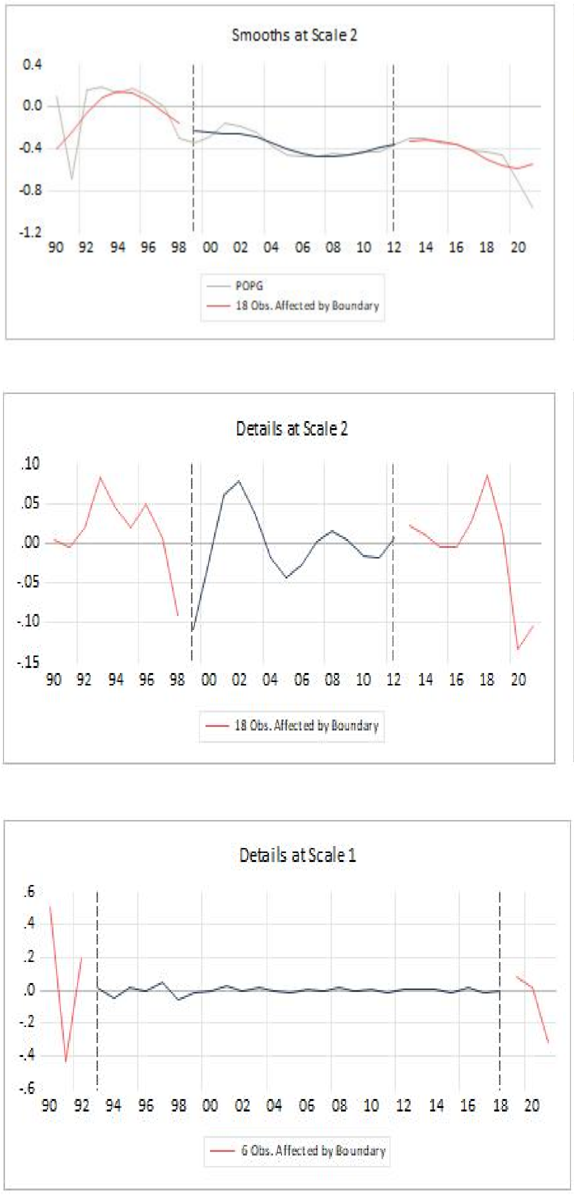


Figure 6. Bulgarian POPG: MODWT

Source: Author’s design based on UN data

It is worth considering that Multi resolution analysis (MRA) is often used as an in-between step toward some final conclusion procedure (Bilen & Huzurbazar, 2002). The purpose of MRA is to set the maximum decomposition level to a lower value. This is because the smooth series extracts the ‘‘signal’’ from the original series for all scales beyond the maximum decomposition level, while the ‘‘noise’’ part of the original series is decomposed scale-by-scale for all scales up to the maximum decomposition level (IHS, 2022). Therefore, getting rid of noise from regressors may hinder the glooming of conclusions. Figures 7-8 present the MODWT MRA of the POPG using a Daubechies filter of length 4 and maximum decomposition level 2. The output in Figure 7-8 is also a spool object with smooth and detail series as individual spool components. All observations affected by the boundary coefficients are presented in red and their number indicated in the legend. In addition, the observations affected by the boundary are divided between the beginning and end of original series observations by two dotted vertical lines on each decomposition scale. These lines divide the areas that divide the total set of observations into those affected by the boundary, and those that are not.



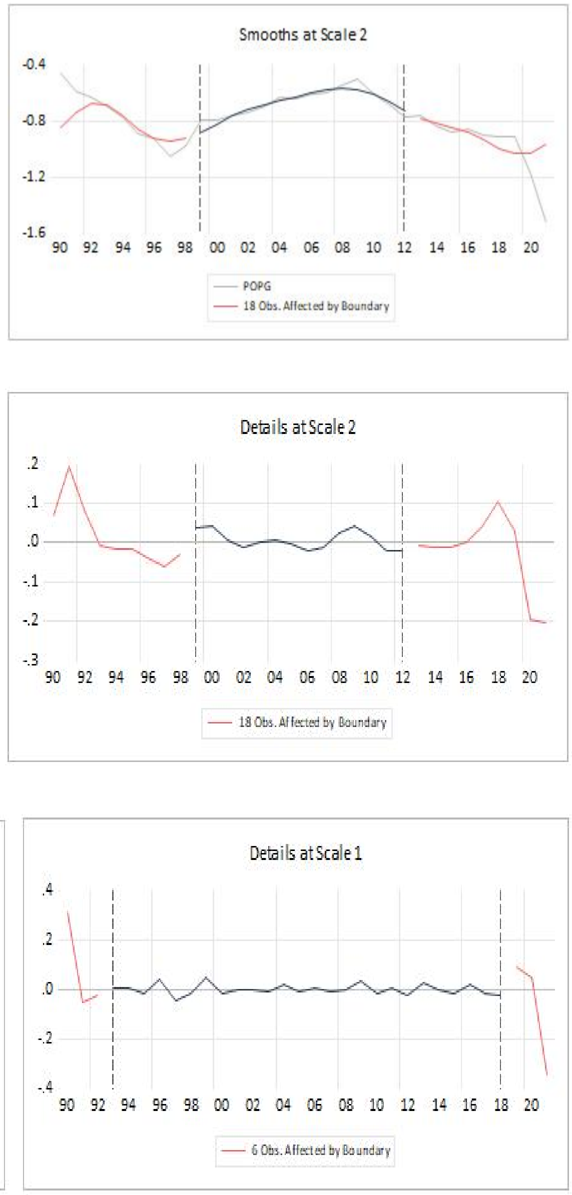


Figure 7. Serbian MODWT MRA Smooth and Details (left)

Figure 8. Bulgarian MODWT MRA Smooth and Details (right)

Source: Author's design based on UN data

Source: Author's design based on UN data

The spool output in Figure 9-10 presents the variance spectrum distribution across-scales, confidence intervals (CIs) across scales, and the cumulative variance. A scale-by-scale decomposition of variance contributions was performed using the MODWT with a Daubechies filter of length 4. Additionally, 95% confidence intervals were generated using the asymptotic Chi-squared distribution with a band-pass estimate because there was small size of the sample. The first graph is a histogram of variances on each specific scale. It is clear that the major part of variation in the POPG series in both countries comes from higher scales, or lower frequencies. This is an indication of persistent behavior in the original data for both countries.

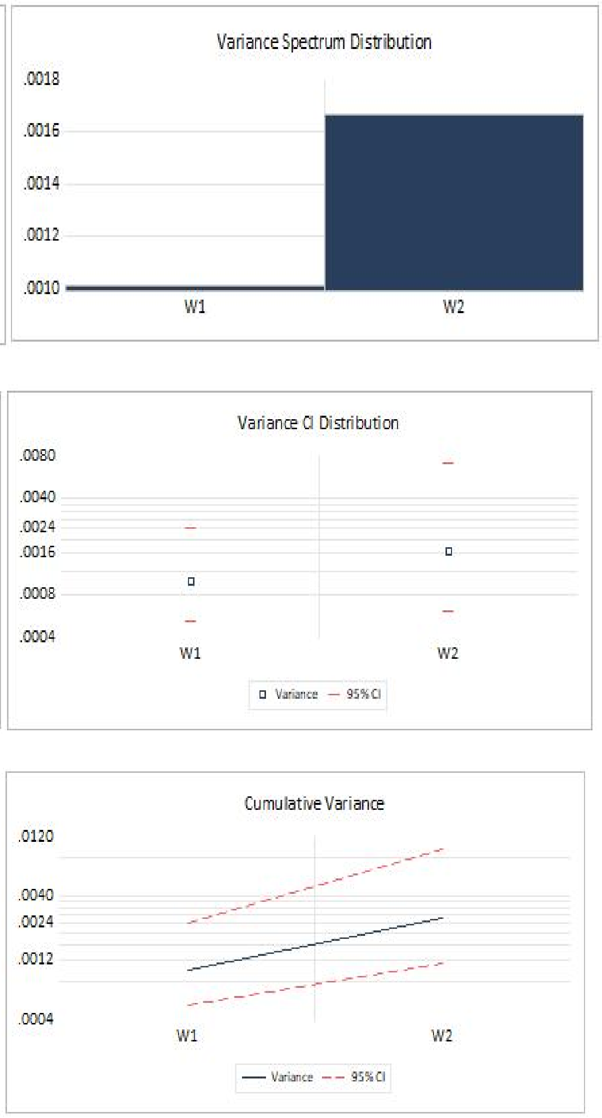


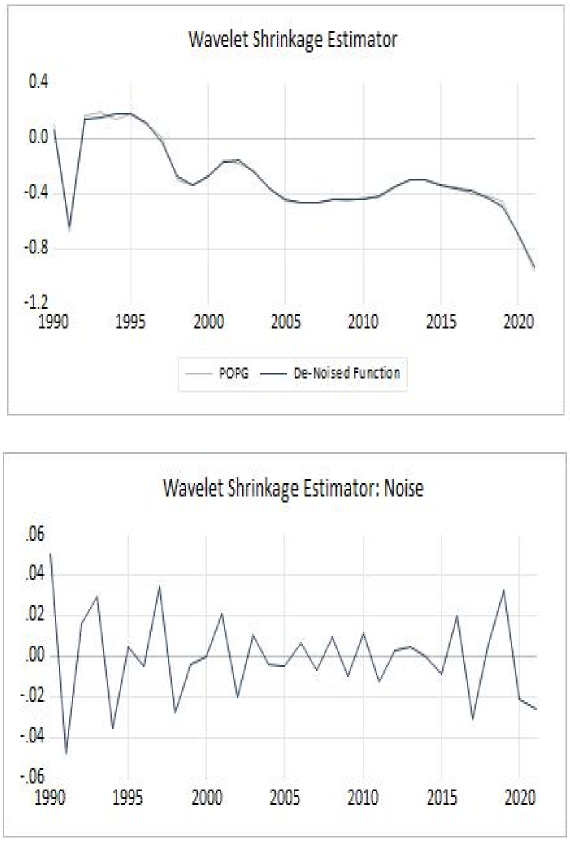
Figure 9. Serbian MODWT Variance decomposition (left)

Figure 10. Bulgarian MODWT Variance decomposition (right)

Source: Author's design based on UN data

Source: Author's design based on UN data

An especially important aspect of empirical work is recognizing useful data from noise. Thus, if there is a doubt that within an observed time series there could be found a presence of unwanted noise, it is crucial to secure an estimate of this noise and filter it from the observed data in order to retain the useful information, or the signal. This is explicitly useful in visualizing which wavelet coefficients should be shrunk to zero, i.e., are insignificant in wavelet shrinkage applications. Put in other words, the coefficients exceeding some threshold bound (in our case the standard deviation) should be retained, while the remaining coefficients are shrunk to zero. A proximate examination of the wavelet threshold noise series reveals moderate differences between the countries. From our results it can be noticed that the majority of wavelet coefficients at scale 1 for both Serbian and Bulgarian POPG can be abandoned and only slightly coefficients can be retained.



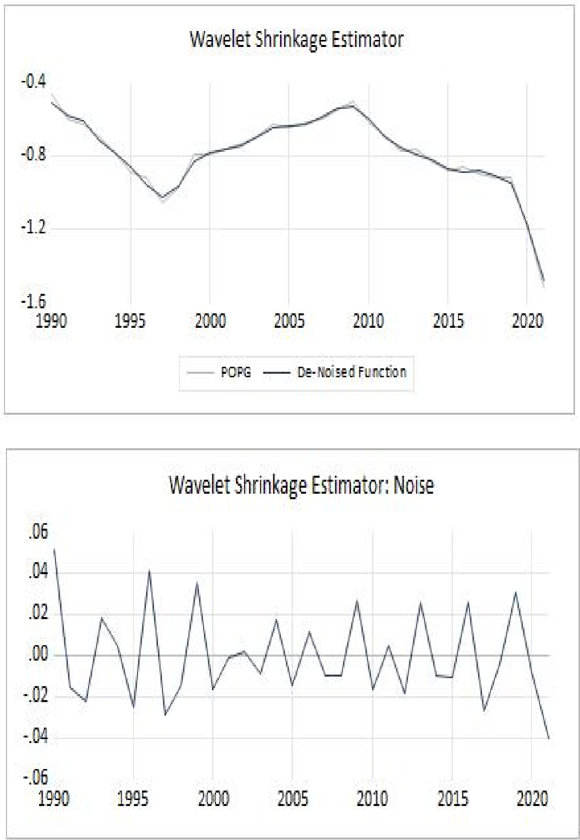


Figure 11. Serbian POPG: MODWT Thresholding **Figure 12.** Bulgarian POPG: MODWT Thresholding

Source: Author's design based on UN data Source: Author's design based on UN data

Also, particularly practical and of great significance in application of wavelets is outlier detection. The idea behind is that extreme (outlying) values will register as apparent spikes in the spectrum. As such, these values would be considered for outlying observations. Obviously, the large outlying observation in 1990 and 2020 is accurately identified for the Serbian POPG. In addition, there is one more outlying observation identified in 2020 for Bulgarian POPG. Since 2020 was a COVID-19 year, it is very clear why this year is an outlier for both countries since both countries lost many lives due to the COVID-19 crisis. For Serbia, 1990 is also an outlier and this is so because this year is a turning point for Serbia after a breakup was hinted at soon, i.e., in 1991, the former Yugoslavian federation, of which Serbia was a part, officially fell apart.

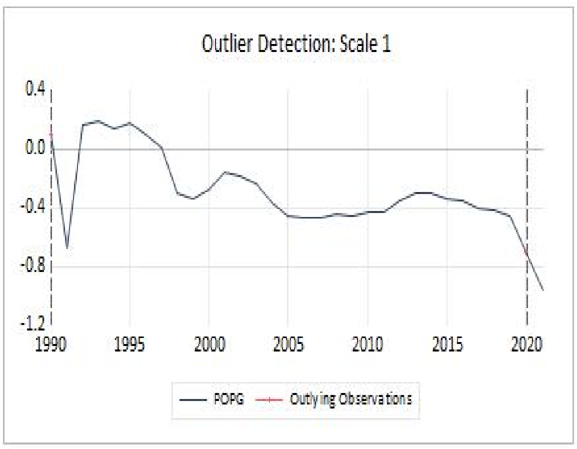


Figure 13. Serbian POPG: Outlier detection

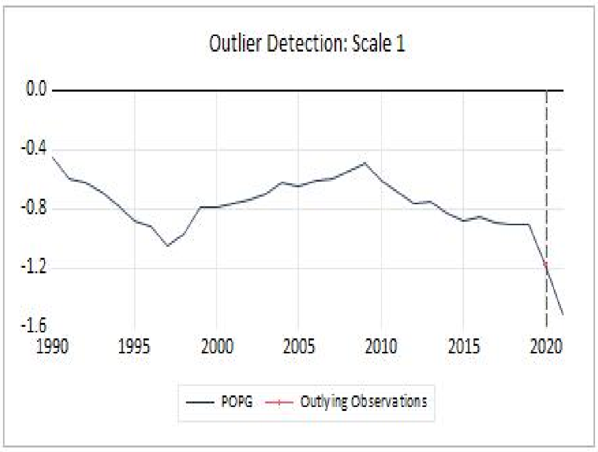


Figure 14. Bulgarian POPG: Outlier detection

Source: Author's design based on UN data

Source: Author's design based on UN data

5. Conclusions

As a mathematical tool wavelet analysis has been in use for analysis for a long time. In this research work it was demonstrated how the theoretical principles in wavelet analysis are applied to real population growth data using the new wavelet engine. It is undoubtedly that our study provides further contribution for the population growth measures. There were defined a 32-objective measure of population growth rate by analyzing the temporal patterns of 32 population growth rates in two countries which have had a predominantly negative population growth.

Those wavelet coefficients that exceeded the threshold bounds are the coefficients which contribute most to explaining variation at that scale and may catch the most transitory (noise) features of the POPG series for Serbia and Bulgaria. It is easy to understand that the major part of relevant wavelet coefficients (those coefficients exceeding threshold bounds) are concentrated in the first few scales. It is also noticed that there are significantly more coefficients of this type in the MODWT transformation in contrast to the DWT. This behavior can be seen also in the wave-models in the spool output. The MRA is usually used to obtain approximations for the original series using their lower and upper frequency components. The upper frequency components are grasped by the wavelet coefficients and therefore connected with features lasting for a short-time, i.e., are transitory in nature. Thus, the upper frequency components of the POPG original series which usually are related with "noise" are extracted by the detail series. On the other hand, lower frequency components are grasped by the scaling coefficients and therefore related with features that are never changing or are long-term in nature. Therefore, these features of the POPG original series which are usually known as the "signal" are extracted by the smooth series. In addition, as wavelet filtering could result in boundary coefficients then the detail as well as smooth POPG series for both countries also have observations affected by the boundary.

For a univariate time, series surroundings, economic turbulences, such as the recent economic crises or and inflations should be directly related with higher wavelet power as the wavelet power spectrum reflects the variance of the time series. For POPG series in both countries lower frequencies were associated with larger contributions to overall variation. This was an indication of persistent behavior in POPG for both countries during this period of analysis. The wavelet analysis for Serbia and Bulgaria show further that economic shocks are predominantly long-lived, which implies that there is no sudden change of trends in POPG. A fast look at the POPG series for both countries indicates that there is an uncommonly large drop in 2020, falling very sharp. This is an unusually great drop and is almost definitely an outlying observation. This outlier was identified as a result of the COVID-19 crises.

However, it was also shown that a significant temporal evolution is present in the slope of the relationship in the data series. Finally, our research results from the wavelet analysis suggest that there could be provided a relevant theoretical overview in understanding of the local circumstances and significant factors regarding population growth in both countries separately during this period of three decades. Considering the results of the empirical analysis, the theoretical approach of Robinson and Srinivasan (1997) can be generally accepted. Thus, these scholars understand that the evolution of population, technology and utilization of resources as well as government policymaking and social institutions can be considered as jointly endogenous outcomes resulting from the decisions of agents in an economy. In addition, the postulates of the endogenous growth theory in Mohamed and Schachler (2016) are also reflected within our findings. Furthermore, close to our findings is the multi-trajectory model developed by Read and LeBlanc (2003). What really connects our research with this model is the complex relationship found between population growth and societal change. The understanding that different societies and cultures have followed different dynamic trajectories is in line with the comprehension of the changes in population growth dynamics over a period of time.

Author's Contribution

The author is solely responsible for the full

content of this research work.

JEL Codes: C01, C22, C23, C87, E02

References

Avdeev, A., Eremenko, T., Festy, P., Gaymu, J., Le Bouteillec, N., and Springer, S. (2011). Populations and Demographic trends of European countries, 1980-2010. *Population, 66,* 9-133. doi: 10.3917/popu.1101.0009.

Bašta, M. (2011). The Modwt Analysis of the Relationship Between Mortality and Ambient Temperature for Prague, Czech Republic. *Acta Oeconomica Pragensia*, *19*(1), 20-40. doi:10.18267/j.aop.324.

Bilen, C., and Huzurbazar, S. (2002). Wavelet-based detection of outliers in time series. *Journal of Computational and Graphical Statistics*, *11*(2), 311-327.

Bloom, D.E., and Canning, D. (2006). Booms, Boosts, and Echoes. F*inance & Development, A quarterly magazine of the IMF,* September 2006, *43*(3). Online available at: Booms, Busts, and Echoes - Finance & Development, September 2006 (imf.org).

Bremner, J., Lopez-Carr, D.L., Suter, L., Davis, J. (2010). Population, poverty, environment, and climate dynamics in the developing world. *Interdisciplinary Environmental Review*, *11*(2-3), 112-126.

Cazelles, B., Cazelles, K., Chavez, M. (2014). Wavelet analysis in ecology and epidemiology: impact of statistical tests. *J. R. Soc. Interface*, *11*. http://dx.doi.org/10.1098/rsif.2013.0585.

Conlon, T., Cotter, J., and Gençay, R. (2018) Long-run wavelet-based correlation for financial time series. *European Journal of Operational Research*, *271*(2), 676-696. https://doi.org/10.1016/j.ejor.2018.05.028.

Conlon, T., and Cotter, J. (2011). An Empirical Analysis of Dynamic Multiscale Hedging Using Wavelet Decomposition. *Journal of Futures Markets*, *32*(3), 272-299. Available at SSRN: https://ssrn.com/abstract=1756800 or http://dx.doi.org/10.2139/ssrn.1756800.

Conlon, T., Corbet, S., and McGee, R.J. (2021). Inflation and crypto currencies revisited: A time-scale analysis. *Economics Letters, 206*(C). https://doi.org/10.1016/j.econlet.2021.109996

Djurdjev, B., and Arsenović, D. (2015). Population dynamics in the intercense period. In Nikitovic, V. (ed.), *The Population of Serbia at the beginning of the 21st century*. RZS, Belgrade, pp. 42-60.

IHS Global Inc. (2020). *EViews 12 Getting Started*. IHS Markit, Seal Beach: CA.

IHS Global Inc. (2022). *EViews 13 User's Guide II*. IHS Markit, Seal Beach: CA.

Klarl, T. (2016). The nexus between housing and GDP re-visited: A wavelet coherence view on housing and GDP for the U.S. *Economics Bulletin*, 36(2), 704-720.

Lanzieri, G. (2010). Looking 50 years ahead: a 'natural' projection of the populations of the Balkan countries to 2061. IV International Conference of Balkans Demography, Budva, Montenegro, 13-15 May 2010.

Lanzieri, G. (2013). Long-term contribution of migration in ageing populations: Japan compared with Europe. Eurostat, Statistical working papers.

Lesthaeghe, R., and Surkyn, J. (2021). The notion of the negative population growth momentum illustrated with Italian data. Interface Demography, Free University of Brussels (VUB). Text: 11/1/2021.

Lee, R. (2011). The outlook for population growth. *Science*, *333*(6042), 569-573. DOI: 10.1126/science.1208859.

Marsiglio, S. (2010). Endogenous growth, population growth and the repugnant conclusion. Working Paper 2010-14, April 2010. University of Milan, Department of Economics.

Merry, R.J.E., and Steinbuch, M. (2005). Wavelet theory and applications. Literature study, Eindhoven University of Technology, Department of Mechanical Engineering, Control Systems Technology Group.

Mohamed, G.G.A., and Schachler, M.H. (2016). Population growth and transitional dynamics of Egypt Theoretical analysis & Time series analysis from 1981 to 2007. *International Journal of Asian Social Science*, *7*(2), 110-118. DOI: 10.18488/journal.1/2017.7.2/1.2.110.118.

Palivos, T., and Yip, C.K. (1993). Optimal population size and endogenous growth. *Economics Letters*, *41*, 107-110.

Pnevmatikos, N.G. (2010). Damage Detection of structures using discrete Wavelet transform. Fifth World Conference on Structural Control and Monitoring (5WCSCM), 12-14

July 2010 Tokyo, Japan, CD Proceedings, paper #35.

Peretto, F.P. (1998). Technological Change and Population Growth. *Journal of Economic Growth*, *3*, 283–311.

Prskawetz, A., Feichtinge, G. (1995) Endogenous population growth may imply chaos. *Journal of Population Economics*, *8*, 59-80.

Porwik, P., and Lisowska, A. (2004). The Haar–Wavelet Transform in Digital Image Processing: Its Status and Achievements. *Machine Graphics & Vision*, *13*(1-2), 79-98.

Robinson, J.A., and Srinivasan, T.N. (1997). Long-term consequences of population growth: technological change, natural resources, and the environment. In Rosenzweig, M.R., and Stark, O.(eds.), *Handbook of Population and Family Economics*. Elsevier Science B.V., Amsterdam, pp. 1175-1298.

Read, D.W., and LeBlanc, S.A. (2003) Population Growth, Carrying Capacity, and Conflict. *Current Anthropology*, *44*(1), 59-85.

Schoen, R. (2018). The time to decline: tracing a cohort's descendants in below replacement populations. *Genus*, *74*(1), 1-10. doi: 10.1186/s41118-018-0026-x.

Singh, G.S.P., and Loh, L. (2010). Wavelet perspective of time varying dynamic of Chinese, Indian and other major markets. *Investment Management and Financial Innovations, 7*(2), 100-107.

United Nations. (2022). World Population Prospects 2022. Department of Economic and Social Affairs, Population Division. Available from: https://population.un.org/wpp [Last accessed on 2022 Dec 29].

Velarde, G., Meredith, D., and Weyde, T. (2016). A Wavelet-Based Approach to Pattern Discovery in Melodies. In Meredith D (ed.), *Computational Music Analysis. Springer*, Cham, pp 303-333. DOI: 10.1007/978-3-319-25931-4_12.

[1] A wavelet is a small wave which becomes larger and larger and then decomposes in a limited time period.

[2] A dyadic network discretizes the scale parameter on a logarithmic scale. The time parameter is discretized with regard to the scale parameter. The dyadic network is one of the most clear and efficient discretization methods for applicative aims and leads to the building of both orthogonal and normalized wavelet basis (Merry and Steinbuch, 2005).