

Light Clock Behavior Explained via Relativistic Aberration of Light

A. Sfarti¹

¹ 387 Soda Hall, UC Berkeley, Berkeley, USA

Correspondence: A. Sfarti, 387 Soda Hall, UC Berkeley, Berkeley, USA.

doi:10.56397/JPEPS.2023.12.03

Abstract

In the following paper we explain the functionality of the light clock via the relativistic aberration of light. A second explanation, through relativistic speed composition will be provided. The behavior of the light clock is an important tool in explaining time dilation. This article is intended for educators teaching special relativity as well as for students.

Keywords: light clock, relativistic aberration, relativistic speed composition

1. Introduction

The behavior of the light beam inside the so-called “light clock” is a source of puzzlement for generations of students studying special relativity. Why does the light (see Figure 1) follow a zig-zag pattern in a frame moving with respect to the clock? What is the relationship between the shape of the zig-zag pattern and the speed v between the observer and the clock? In the following, we will give two different explanations of what is going on.

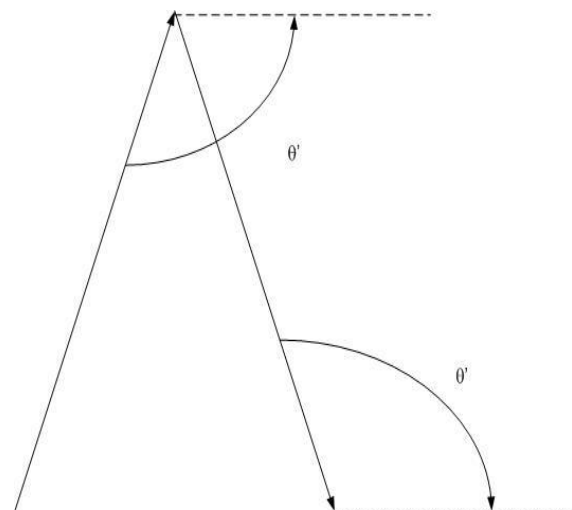


Figure 1. The light clock viewed by an observer in motion

2. Solution by Application of the Relativistic Aberration of Light

Looking at figure 1 we can show that the angle made by the “up” going light beam with the horizontal is given by the light aberration formula (A. Einstein, 1905):

$$\cos \theta' = \frac{\cos \theta - \frac{v}{c}}{1 - \frac{v}{c} \cos \theta} = -\frac{v}{c} \quad (2.1)$$

On the “downward” going light beam we can write just the same:

$$\cos \theta' = \frac{\cos \theta - \frac{v}{c}}{1 - \frac{v}{c} \cos \theta} = -\frac{v}{c} \quad (2.2)$$

The above fully explain the zig-zag shape of the path followed by the light beam as seen by an observer moving at speed v alongside the x axis with respect to the clock. The time dilation between the frame commoving with the clock and the frame commoving with the observer is derived immediately from Pythagoras theorem:

$$(ct)^2 = (ct')^2 - (vt')^2 \quad (2.3)$$

From (2.3) the time dilation formula follows immediately:

$$t = t' \sqrt{1 - \frac{v^2}{c^2}} \quad (2.4)$$

We see how the relativistic aberration of light has provided an elegant explanation of the light clock behavior.

3. Solution by Application of the Relativistic Speed Composition

In the frame commoving with the clock the light velocity has the components $(c_x, c_y) = (0, c)$.

In the frame of the observer moving with respect to the clock, the velocity has the components (A. Sfarti, 2017; 2014):

$$\begin{aligned} c'_x &= \frac{c_x - v}{1 - \frac{c_x v}{c^2}} = -v \\ c'_y &= \frac{c}{\gamma} = c \sqrt{1 - \frac{v^2}{c^2}} \end{aligned} \quad (3.1)$$

The angle made by the light beam with respect to the horizontal, as measured in the frame of the observer is:

$$\cos \theta' = \frac{c'_x}{\sqrt{c'^2_x + c'^2_y}} = -\frac{v}{c} \quad (3.2)$$

Another interesting fact is that the light speed along the inclined path, in the frame of the observer is;

$$\sqrt{c'^2_x + c'^2_y} = c \quad (3.3)$$

4. Conclusions

We have explained the behavior of the relativistic light clock through two different ways, providing some simple and robust solutions.

References

- A. Einstein. (1905). On the Electrodynamics of Moving Bodies. *Ann. Der Phys.*
- A. Sfarti. (2014). Comment on the Paper Absolute Motion Determined from Michelson-Type Experiments in Optical Media: by V.P. Dmitriyev. *Zeit. Fur Nat. A*, 66a(12).
- A. Sfarti. (2017). General Clocks and the Clock Hypothesis. *Theoretical Physics*, 2(4).