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Relativistic Dynamics and Electrodynamics in Uniformly Accelerated and in Uniformly Rotating Frames—the Proper Time Expressions

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Abstract

In the current paper we present a generalization of the transforms from the frame co-moving with an accelerated particle, for either rectilinear or circular motion, into an inertial frame of reference. The solution is of great interest for real time applications, because earth-bound laboratories are inertial only in approximation. The motivation is that the real-life applications include accelerating and rotating frames with arbitrary orientation more often than the idealized case of inertial frames; our daily experiments happen in the laboratories attached to the rotating Earth. Our paper is divided into two main sections, the first section deals with dynamics, i.e., forces, the second section deals with electromagnetism, i.e., Lorentz force and electromagnetic potentials, both expressed in terms of proper time.

Keywords: accelerated motion, general coordinate transformations, accelerated particles, uniform rotation, four-force vector, four-electromagnetic vector, Lorentz force, proper time

1. Introduction

Real life applications include accelerating and rotating frames more often than the idealized case of inertial frames. Our daily experiments happen in the laboratories attached to the rotating Earth. Many books and papers have been dedicated to transformations between particular cases of rectilinear acceleration and/or rotation (Moller, C., 1960) and to the applications of such formulas (Thomas, L. H., 1926; Ben-Menahem, A., 1985; Ben-Menahem, S., 1986; Kroemer, H., 2004; Rhodes, J. A., Semon, M. D., 2005; Malykin, G. B., 2006; Krivoruchenko, M. I., 2009; Sfarti, A., 2009; Sfarti, A., 2010; Sfarti, A., 2012; Rebilas, K., 2013; Nelson, R. A., 1987; Sfarti, A., 2016). There is great interest in producing a general solution that deals with arbitrary orientation of acceleration in the case of rectilinear motion and for arbitrary direction of uniform angular velocity.

The main idea of this paper is to generate a standard blueprint for a general solution. The blueprint relies on transforming the problem geometrically in the "canonical reference frame" of (Moller, C., 1960), followed by the application of the physical transforms derived for such "canonical" orientations (Moller, C., 1960; Thomas, L. H., 1926; Ben-Menahem, A., 1985; Ben-Menahem, S., 1986; Kroemer, H., 2004; Rhodes, J. A. & Semon, M. D., 2005; Malykin, G. B., 2006) and ending with the application of the inverse geometrical transformation:

$$Geometry\_Transform -> Physics\_Transform -> Inverse\_Geometry\_Transform \tag{1.1}$$

We conclude our paper with a practical application of deriving the formula of the Lorentz force in a uniformly rotating frame.

2. Dynamics in Accelerated Rectilinear Motion

Let S represent an inertial system of coordinates and $S'(\tau)$ an accelerated one. Moller (1960) considers a particular case where a particle moves with acceleration $\mathbf{g} = (g, 0, 0)$ aligned with the x-axis. According to reference (Moller, C., 1960) the general transformation for the particular case from $S'(\tau)$ into S is:

$$\begin{pmatrix} x \\ y \\ z \\ ct \end{pmatrix} = \mathbf{Phy\_rectilinear} \begin{pmatrix} x' \\ y' \\ z' \\ ct' \end{pmatrix} \tag{2.1}$$

where:

$$\mathbf{Phy\_rectilinear} = \begin{bmatrix} \cosh\frac{g\tau}{c} & 0 & 0 & \sinh\frac{g\tau}{c} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sinh\frac{g\tau}{c} & 0 & 0 & \cosh\frac{g\tau}{c} \end{bmatrix} \tag{2.2}$$

$$\frac{d}{d\tau}\begin{pmatrix} x \\ y \\ z \\ ct \end{pmatrix} = \mathbf{Phy\_rectilinear}\frac{d}{d\tau}\begin{pmatrix} x' \\ y' \\ z' \\ ct' \end{pmatrix} + \frac{d\mathbf{Phy\_rectilinear}}{d\tau}\begin{pmatrix} x' \\ y' \\ z' \\ ct' \end{pmatrix} \tag{2.3}$$

$$\frac{d\mathbf{Phy\_rectilinear}}{d\tau} = \frac{g}{c}\begin{bmatrix} \sinh\frac{g\tau}{c} & 0 & 0 & \cosh\frac{g\tau}{c} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \cosh\frac{g\tau}{c} & 0 & 0 & \sinh\frac{g\tau}{c} \end{bmatrix}$$

$$\begin{aligned} \frac{dt}{d\tau} &= \cosh\frac{g\tau}{c} \\ \frac{dt'}{d\tau} &= \cosh\frac{g\tau}{c} \end{aligned} \tag{2.4}$$

The speed measured in the inertial frame S depends both on the speed and the position measured in the accelerated frame S′.

In the following section we generalize his derivation for the arbitrary case $\mathbf{g} = (g_x, g_y, g_z)$ for obtaining the general four-space coordinate transformations that take us from $S'(\tau)$ into S. Expressed in polar coordinates, the acceleration has the form:

$$\begin{aligned} g_x &= g\cos\theta\cos\varphi \\ g_z &= g\sin\theta\cos\varphi \\ g_y &= g\sin\varphi \end{aligned} \tag{2.5}$$

$$\varphi = \arcsin\frac{g_y}{g}$$

$$\theta = \arctan\frac{g_z}{g_x}$$

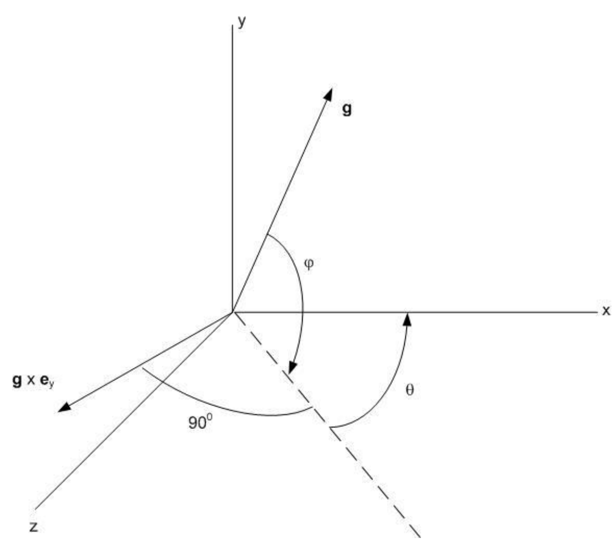

Figure 1. Arbitrary direction rectilinear accelerated motion

The first step rotates the unit vector of acceleration $\mathbf{g}$ by $90^0 - \varphi$ around the axis the vector cross-product $\mathbf{g}\times\mathbf{e}_y = -\frac{g_z}{g}\mathbf{e}_x + \frac{g_x}{g}\mathbf{e}_z$ such $\mathbf{g}$ gets aligned with the y-axis (see Fig.1). For this purpose, we will introduce the triplet $(a,b,c) = (-\frac{g_z}{g}, 0, \frac{g_x}{g})$. The following expressions hold (Paul Bourke, 1992):

$$\mathbf{Rot_y} = \begin{bmatrix} c & 0 & -a & 0 \\ 0 & 1 & 0 & 0 \\ a & 0 & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.6)$$

$$\mathbf{Rot(e_y)_{90^0-\varphi}} = \begin{bmatrix} \cos(90^0-\varphi) & 0 & -\sin(90^0-\varphi) & 0 \\ 0 & 1 & 0 & 0 \\ \sin(90^0-\varphi) & 0 & \cos(90^0-\varphi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sin\varphi & 0 & -\cos\varphi & 0 \\ 0 & 1 & 0 & 0 \\ \cos\varphi & 0 & \sin\varphi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.7)$$

$\mathbf{Rot(e_y)_{90^0-\varphi} * Rot_y}$ aligns $\mathbf{g}$ with $\mathbf{e}_y$. The second step is comprised by another rotation around the z-axis by -90^0 that aligns $\mathbf{g}$ with $\mathbf{e}_x$:

$$\mathbf{Rot(e_z)_{-90^0}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.8)$$

Putting it all together:

$$\mathbf{Rr = Rot(e_x)_{90^0} * Rot(e_y)_{90^0-\varphi} * Rot_y} \quad (2.9)$$

The general coordinate transformation between S and $S'(\tau)$ is:

$$\begin{pmatrix} x \\ y \\ z \\ ct \end{pmatrix} = \mathbf{A} \begin{pmatrix} x' \\ y' \\ z' \\ ct' \end{pmatrix} \quad (2.10)$$

where:

$$\mathbf{Rr^{-1} * Phy\_rectilinear * Rr} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = \mathbf{A} \quad (2.11)$$

The general velocity transformation is therefore:

$$\begin{pmatrix} v_x \\ v_y \\ v_z \\ c\cosh\frac{g\tau}{c} \end{pmatrix} = \mathbf{A} \begin{pmatrix} v_x' \\ v_y' \\ v_z' \\ c\cosh\frac{g\tau}{c} \end{pmatrix} + \frac{d\mathbf{A}}{d\tau} \begin{pmatrix} x' \\ y' \\ z' \\ ct' \end{pmatrix} \quad (2.12)$$

where:

$$\frac{d\mathbf{A}}{d\tau} = \mathbf{Rr^{-1}} * \frac{d\mathbf{Phy\_rectilinear}}{d\tau} * \mathbf{Rr} \quad (2.13)$$

The inverse transform is:

$$\begin{pmatrix} v_x' \\ v_y' \\ v_z' \\ c\cosh\frac{g\tau}{c} \end{pmatrix} = \mathbf{A}^{-1} \begin{pmatrix} v_x \\ v_y \\ v_z \\ c\cosh\frac{g\tau}{c} \end{pmatrix} + \frac{d\mathbf{A}^{-1}}{d\tau} \begin{pmatrix} x \\ y \\ z \\ ct \end{pmatrix} \quad (2.14)$$

where:

$$\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} = \mathbf{A}^{-1} = \mathbf{Rr^{-1} * Phy\_rectilinear^{-1} * Rr} \quad (2.15)$$

The energy-momentum transforms the same way as the 4-coordinates (2.10) by virtue of being a 4-vector:

$$\begin{pmatrix} p_x \\ p_y \\ p_z \\ E/c \end{pmatrix} = \mathbf{A} \begin{pmatrix} p_x' \\ p_y' \\ p_z' \\ E'/c \end{pmatrix} \quad (2.16)$$

Therefore:

$$\begin{pmatrix} F_x \\ F_y \\ F_z \\ \frac{1}{c}\frac{dE}{d\tau} \end{pmatrix} = \mathbf{A}\begin{pmatrix} F_x^{'} \\ F_y^{'} \\ F_z^{'} \\ \frac{1}{c}\frac{dE'}{d\tau} \end{pmatrix} + \frac{d\mathbf{A}}{d\tau}\begin{pmatrix} p_x^{'} \\ p_y^{'} \\ p_z^{'} \\ E'/c \end{pmatrix} \quad (2.17)$$

The inverse transform, from the inertial frame S into the accelerated frame S', is:

$$\begin{pmatrix} F_x^{'} \\ F_y^{'} \\ F_z^{'} \\ \frac{1}{c}\frac{dE'}{d\tau} \end{pmatrix} = \mathbf{A}^{-1}\begin{pmatrix} F_x \\ F_y \\ F_z \\ \frac{1}{c}\frac{dE}{d\tau} \end{pmatrix} + \frac{d\mathbf{A}^{-1}}{d\tau}\begin{pmatrix} p_x \\ p_y \\ p_z \\ E/c \end{pmatrix} \quad (2.18)$$

3. Dynamics in Uniform Angular Velocity Rotation

In this section we discuss the case of the particle moving in an arbitrary plane, with the normal given by the constant angular velocity $\boldsymbol{\omega}(a,b,c)$ (see Figure 2). According to Moller (1960), the simpler case when $\boldsymbol{\omega}$ is aligned with the z-axis produces the transformation between the rotating frame $S'(\tau)$ attached to the particle and an inertial, non-rotating frame S attached to the center of rotation:

$$\begin{pmatrix} x \\ y \\ z \\ ct \end{pmatrix} = \mathbf{Phy\_rotation}\begin{pmatrix} x' \\ y' \\ z' \\ ct' \end{pmatrix} \quad (3.1)$$

where:

$$\mathbf{Phy\_rotation} = \begin{bmatrix} \cos\alpha\cos\beta+\gamma\sin\alpha\sin\beta & \sin\alpha\cos\beta-\gamma\cos\alpha\sin\beta & 0 & -\frac{u\gamma}{c}\sin\beta \\ \cos\alpha\sin\beta-\gamma\sin\alpha\cos\beta & \sin\alpha\sin\beta+\gamma\cos\alpha\cos\beta & 0 & \frac{u\gamma}{c}\cos\beta \\ 0 & 0 & 1 & 0 \\ \frac{u\gamma}{c}\sin\alpha & -\frac{u\gamma}{c}\cos\alpha & 0 & \gamma \end{bmatrix}$$

(3.2)

$$\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$$
$$u = r\omega$$
$$\alpha = \omega\gamma\tau$$
$$\beta = \omega\gamma^2\tau \quad (3.3)$$

The general case is treated by transforming the problem into the particular case treated in (Moller, C., 1960) through a transformation into the "canonical case", followed by an application of the transformation from the accelerated frame into the inertial frame, ending with the inverse of the first transformation, as shown below:

$$\begin{pmatrix} x \\ y \\ z \\ ct \end{pmatrix} = (\mathbf{Rr}^{-1} * \mathbf{Phy\_rotation} * \mathbf{Rr})\begin{pmatrix} x' \\ y' \\ z' \\ ct' \end{pmatrix} = \mathbf{A}\begin{pmatrix} x' \\ y' \\ z' \\ ct' \end{pmatrix}$$

(3.4)

$$\mathbf{Rr} = \mathbf{Rot(e_x)}_{90^0} * \mathbf{Rot(e_y)}_{90^0-\varphi} * \mathbf{Rot_y} \quad (3.5)$$

$\mathbf{Rot(e_y)}_{90^0-\varphi} * \mathbf{Rot_y}$ aligns $\boldsymbol{\omega}$ with $\mathbf{e}_y$. The second step is comprised by another rotation around the x-axis by 90^0 that aligns $\boldsymbol{\omega}$ with $\mathbf{e}_z$:

$$\mathbf{Rot(e_x)}_{90^0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.6)$$

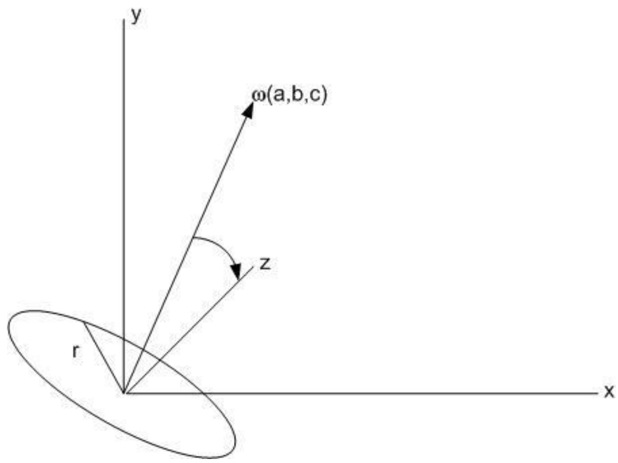

Figure 2. Uniform rotation with arbitrary direction of angular velocity

Expression (3.4) gives the solution for the general case, of arbitrary angular velocity direction.

The general velocity transformation is:

$$\begin{pmatrix} v_x \\ v_y \\ v_z \\ \frac{c}{\sqrt{1-(\frac{r\omega}{c})^2}} \end{pmatrix} = \mathbf{A} \begin{pmatrix} v_x' \\ v_y' \\ v_z' \\ \frac{c}{\sqrt{1-(\frac{r\omega}{c})^2}} \end{pmatrix} + \frac{d\mathbf{A}}{d\tau} \begin{pmatrix} x' \\ y' \\ z' \\ ct' \end{pmatrix} \quad (3.7)$$

The general force transformation is:

$$\begin{pmatrix} F_x \\ F_y \\ F_z \\ \frac{1}{c}\frac{dE}{d\tau} \end{pmatrix} = \mathbf{A} \begin{pmatrix} F_x' \\ F_y' \\ F_z' \\ \frac{1}{c}\frac{dE'}{d\tau} \end{pmatrix} + \frac{d\mathbf{A}}{d\tau} \begin{pmatrix} p_x' \\ p_y' \\ p_z' \\ E'/c \end{pmatrix} \quad (3.8)$$

where:

$$\mathbf{A} = \mathbf{Rr^{-1} * Phy\_rotation * Rr} \quad (3.9)$$

The reverse transformation is:

$$\begin{pmatrix} F_x' \\ F_y' \\ F_z' \\ \frac{1}{c}\frac{dE'}{d\tau} \end{pmatrix} = \mathbf{A}^{-1} \begin{pmatrix} F_x \\ F_y \\ F_z \\ \frac{1}{c}\frac{dE}{d\tau} \end{pmatrix} + \frac{d\mathbf{A}^{-1}}{d\tau} \begin{pmatrix} p_x \\ p_y \\ p_z \\ E/c \end{pmatrix} \quad (3.10)$$

where:

$$\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} = \mathbf{A}^{-1} = \mathbf{Rr^{-1} * Phy\_rotation^{-1} * Rr}$$

(3.11)

We will use (3.10) in the next section, an application that determines the expression of the Lorentz force in a rotating frame.

4. Application—The Expression of the Lorentz Force in a Uniformly Rotating Frame

Assume that we have a particle of charge q and mass m moving in the x-y plane under the influence of a magnetic field $\mathbf{B}$ aligned with the z axis. We know that in the frame of the lab, the expression of the Lorentz force acting on the particle is:

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} = q \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r\omega\cos(\omega t) & r\omega\sin(\omega t) & 0 \\ 0 & 0 & B \end{bmatrix}$$

(4.1)

We would like to find out the expression of the force in the frame co-rotating with the charged particle. For this purpose, we will resort to (3.10).

$$\begin{pmatrix} F_x' \\ F_y' \\ F_z' \\ \frac{1}{c}\frac{dE'}{d\tau} \end{pmatrix} = \mathbf{A}^{-1} \begin{pmatrix} F_x \\ F_y \\ F_z \\ \frac{1}{c}\frac{dE}{d\tau} \end{pmatrix} + \frac{d\mathbf{A}^{-1}}{d\tau} \begin{pmatrix} p_x \\ p_y \\ p_z \\ E/c \end{pmatrix} \quad (4.2)$$

We know from (Sfarti, A., 2016; Sfarti. A., 2009) that:

$$\begin{pmatrix} x \\ y \\ z \\ ct \end{pmatrix} = \begin{pmatrix} r\cos(\omega t) \\ r\sin(\omega t) \\ 0 \\ ct \end{pmatrix} \quad (4.3)$$

$$\begin{aligned} v_x &= -r\omega\sin(\omega t) \\ v_y &= r\omega\cos(\omega t) \quad (4.4) \\ v_z &= 0 \end{aligned}$$

$$p_x = -\frac{mr\omega \sin(\omega t)}{\sqrt{1-(\frac{r\omega}{c})^2}}$$

$$p_y = \frac{mr\omega \cos(\omega t)}{\sqrt{1-(\frac{r\omega}{c})^2}} \tag{4.5}$$

$$p_z = 0$$

$$E = \frac{mc^2}{\sqrt{1-(\frac{r\omega}{c})^2}}$$

$$F_x = qBr\omega \sin(\omega t)$$

$$F_y = -qBr\omega \cos(\omega t) \tag{4.6}$$

$$F_z = 0$$

Substituting (4.3-4.6) into (4.2) we obtain:

$$\begin{pmatrix} F_x^{'} \\ F_y^{'} \\ F_z^{'} \\ \frac{1}{c}\frac{dE'}{d\tau} \end{pmatrix} = \mathbf{A}^{-1} \begin{pmatrix} qBr\omega\sin(\omega t) \\ -qBr\omega\cos(\omega t) \\ 0 \\ 0 \end{pmatrix} + \frac{d\mathbf{A}^{-1}}{d\tau} \begin{pmatrix} -\frac{mr\omega\sin(\omega t)}{\sqrt{1-(\frac{r\omega}{c})^2}} \\ \frac{mr\omega\cos(\omega t)}{\sqrt{1-(\frac{r\omega}{c})^2}} \\ 0 \\ \frac{mc}{\sqrt{1-(\frac{r\omega}{c})^2}} \end{pmatrix} \tag{4.7}$$

To the above, we need to add (Sfarti. A., 2009) the fact that:

$$\omega = \frac{qB}{\gamma(v_0)m}$$

$$r = \frac{\gamma(v_0)mv_0}{qB} \tag{4.8}$$

In (4.8) v_0 is the initial speed of the particle at $t = 0$. Armed with that (4.7) gets the simpler form:

$$\begin{pmatrix} F_x^{'} \\ F_y^{'} \\ F_z^{'} \\ \frac{1}{c}\frac{dE'}{d\tau} \end{pmatrix} = \mathbf{A}^{-1} \begin{pmatrix} qBv_0\sin(\frac{qBt}{\gamma(v_0)m}) \\ -qBv_0\cos(\frac{qBt}{\gamma(v_0)m}) \\ 0 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{1-(\frac{r\omega}{c})^2}}\frac{d\mathbf{A}^{-1}}{d\tau} \begin{pmatrix} -mv_0\sin(\frac{qBt}{\gamma(v_0)m}) \\ mv_0\cos(\frac{qBt}{\gamma(v_0)m}) \\ 0 \\ mc \end{pmatrix} \tag{4.9}$$

5. Electrodynamics in Uniformly Accelerated Frames and in Uniformly Rotating Frames—the General Expressions for the Electromagnetic 4-Vector Potential

Previously (Sfarti, A., 2017; Sfarti, A., 2017) we have dealt with the case of the transformation of Maxwell equations for the case of uniformly accelerated frames and uniformly rotating frames in arbitrary directions. The formalism derived in this paper, section 2, allows us to get a general transformation between the inertial frame S and S' and the inverse. The electromagnetic potential transforms the same way as the 4-coordinates (2.13) by virtue of being a 4-vector:

$$\begin{pmatrix} \Phi_x \\ \Phi_y \\ \Phi_z \\ c\phi \end{pmatrix} = \mathbf{A} \begin{pmatrix} \Phi_x^{'} \\ \Phi_y^{'} \\ \Phi_z^{'} \\ c\phi' \end{pmatrix} \tag{5.1}$$

where $\mathbf{A}$ is given by (2.14) for uniformly accelerated frames and by (3.9) for uniformly rotating frames.

In order to transform Maxwell equations between the frames, we need the **partial** derivatives with respect to x, y, z. We will show how to calculate two of them, as a blueprint.

$$\frac{\partial}{\partial\tau}\begin{pmatrix} \Phi_x \\ \Phi_y \\ \Phi_z \\ c\phi \end{pmatrix} = \mathbf{A}\frac{\partial}{\partial\tau}\begin{pmatrix} \Phi_x^{'} \\ \Phi_y^{'} \\ \Phi_z^{'} \\ c\phi' \end{pmatrix} + \frac{\partial\mathbf{A}}{\partial\tau}\begin{pmatrix} \Phi_x^{'} \\ \Phi_y^{'} \\ \Phi_z^{'} \\ c\phi' \end{pmatrix} \tag{5.2}$$

The inverse transforms are:

$$\frac{\partial}{\partial\tau}\begin{pmatrix} \Phi_x^{'} \\ \Phi_y^{'} \\ \Phi_z^{'} \\ c\phi' \end{pmatrix} = \mathbf{A}^{-1}\frac{\partial}{\partial\tau}\begin{pmatrix} \Phi_x \\ \Phi_y \\ \Phi_z \\ c\phi \end{pmatrix} + \frac{\partial\mathbf{A}^{-1}}{\partial\tau}\begin{pmatrix} \Phi_x \\ \Phi_y \\ \Phi_z \\ c\phi \end{pmatrix} \tag{5.3}$$

By using the above generalized transforms, we can transform the electric and magnetic vectors, thus obtaining the general transformations for the Maxwell equations as seen in (Sfarti, A., 2017; Sfarti, A., 2017).

6. Conclusions

We constructed the general transforms from the frame $S'(\tau)$ co-moving with an accelerated particle for rectilinear or circular motion into an inertial frame of reference S . The solution is of great interest for real life applications, because our earth-bound laboratories are inertial only in approximation; in real life, the laboratories rotate and they are accelerated. We produced a blueprint for generalizing the solutions for the arbitrary case and we concluded with a practical application of deriving the formula of the Lorentz force in a uniformly rotating frame.

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