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Voigt Transformation Not Equivalent to the Lorentz Transformation

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Abstract

In the present paper we dispel the belief that the transformation developed by W. Voigt is equivalent to the Lorentz transformation. Throughout the history, there have been several arguments, both pro and con. In the current paper we present an argument that has not been shown before aimed at settling the dispute. We demonstrate that the Voigt transformation is not equivalent to the Lorentz transformation.

Keywords: special relativity, Voigt transformation, Lorentz transformation, relativistic Doppler effect, relativistic aberration

1. Introduction

In a paper published in 1887, W. Voigt (W. Voigt., 1887) was the first to use the covariance of the propagation equation of the electromagnetic wave to derive the transformation equations that bear his name:

$$\begin{aligned} x' &= x - vt \\ y' &= y / \gamma(v) \\ z' &= z / \gamma(v) \\ t' &= t - \frac{vx}{c^2} \end{aligned} \tag{1}$$

Voigt's program was to re-derive from the above transformation the Doppler effect due to the relative motion between source and observer, an endeavor he accomplished by re-deriving the classical (non-relativistic) effect. That was to be expected since it will be another 18 years before the seminal 1905 Einstein paper (A. Einstein., 1905).

2. What Went Wrong?

Fast forward 17 years and witness Lorentz (H.A. Lorentz., n.d.) deriving the transformation that bears his name starting from the invariance of the propagation equation of the electromagnetic wave (Voigt obtained only the covariance of the equation):

$$\begin{aligned} x' &= \gamma(v)(x - vt) \\ y' &= y \\ z' &= z \\ t' &= \gamma(v)(t - \frac{vx}{c^2}) \end{aligned} \tag{2}$$

While (1) maintains only the covariance of the wave equation, (2) maintains its invariance (R. Heras, n.d.). Lorentz had the advantage of knowing the result of the Michelson-Morley experiment and being familiar with the FitzGerald contraction hypothesis. Transformation (2) enabled Lorentz to re-derive the longitudinal contraction while Voigt transformation (1) does not. If the intent is to re-derive the classical Doppler effect, as in the Voigt program, then using the wave equation as a starting point is a perfect match with the caveat that the derivation can produce either (1) or (2) as a final product. The problem is that the Doppler effect is directional and so is its companion, the aberration so neither of them can be derived from the isotropic form of the wave equations 1″ and 1′ in (W. Voigt., 1887).

3. Einstein's Derivation of Relativistic Doppler Effect and Relativistic Aberration

Einstein (A. Einstein., 1905) after re-deriving the Lorentz transforms directly from his two postulates, proceeds with deriving a lot of interesting consequences, amongst which are the relativistic forms of the Doppler effect and aberration, both of them never seen before. Unlike any of his predecessors, Einstein does not consider relativity as a consequence of electromagnetism but rather a fundamental property of nature. The common starting point for the Doppler and aberration derivation is the invariance of the phase of the planar electromagnetic wave:

$$\Phi = \omega\left(t - \frac{lx + my + nz}{c}\right) \tag{3}$$

Unlike the wave equation, the wave phase has directional content and this directional content will be reflected in both the Doppler effect and in the aberration. Inserting the Lorentz transformation (2) into (3) and using the phase invariance:

$$\Phi = \Phi' = \omega'\left(t' - \frac{l'x' + m'y' + n'z'}{c}\right) \tag{4}$$

Einstein obtains:

$$\begin{aligned}\omega'\left(t' - \frac{l'x' + m'y' + n'z'}{c}\right) &= \omega'\left[\gamma\left(t - \frac{vx}{c^2}\right) - \frac{l'}{c}\gamma(x - vt) - \frac{m'y + n'z}{c}\right] = \\ &= \omega'\gamma\left(1 + \frac{l'v}{c}\right)t - \omega'\gamma\left(-\frac{l'}{c} + \frac{v}{c^2}\right)x - \omega'\frac{m'y + n'z}{c}\end{aligned} \tag{5}$$

The above needs to hold for any (x, y, z, t), so, by identifying the coefficients of the variables between (3) and (5), one obtains the formulas for relativistic Doppler effect and for relativistic aberration:

$$\begin{aligned}\omega &= \omega'\gamma\left(1 + \frac{l'v}{c}\right) \\ l &= \frac{\frac{v}{c} - l'}{1 + \frac{l'v}{c}} \\ m &= \frac{m'}{\gamma\left(1 + \frac{l'v}{c}\right)} \\ n &= \frac{n'}{\gamma\left(1 + \frac{l'v}{c}\right)}\end{aligned} \tag{6}$$

Inserting the Voigt transformation (2) into (3) one obtains:

$$\begin{aligned}\omega'\left(t' - \frac{l'x' + m'y' + n'z'}{c}\right) &= \omega'\left[\left(t - \frac{vx}{c^2}\right) - \frac{l'}{c}(x - vt) - \frac{m'y + n'z}{\gamma c}\right] = \\ &= \omega'\left(1 + \frac{l'v}{c}\right)t - \omega'\left(-\frac{l'}{c} + \frac{v}{c^2}\right)x - \omega'\frac{m'y + n'z}{\gamma c}\end{aligned} \tag{7}$$

By identifying the coefficients of the variables between (3) and (7), one obtains the following consequences of the phase invariance under the Voigt transformations:

$$\omega = \omega'(1+\frac{l'v}{c})$$
$$l = \frac{\frac{v}{c}-l'}{1+\frac{l'v}{c}}$$
$$m = \frac{m'}{\gamma(1+\frac{l'v}{c})} \qquad (8)$$
$$n = \frac{n'}{\gamma(1+\frac{l'v}{c})}$$

Voigt derived only the Doppler effect in his paper. Had he continued with deriving the aberration, he would have found a big surprise: while his derivation recovered the classical (non-relativistic) Doppler effect, the aberration is the relativistic one! This contradiction should have clued him that there was something wrong with his transformation. Today, we know that Voigt transformation would fail the Ives-Stilwell experiment that tests the transverse Doppler effect (G. Saathoff, S. Karpuk, U. Eisenbarth, G. Huber, S. Krohn, R. M. Horta, S. Reinhardt, D. Schwalm, A. Wolf, G. Gwinner, 2003; H. Müller, S. Herrmann, C. Braxmaier, S. Schiller, A. Peters, 2003; H. Müller, S. Herrmann, C. Braxmaier, S. Schiller, A. Peters, 2003; H. Müller, C. Braxmaier, S. Herrmann, A. Peters, & C. Lämmerzahl., 2003). Therefore, according to H.P. Robertson's seminal paper (H.P. Robertson, 1949), Voigt's theory fails the equivalence with special relativity.

Before we conclude this section, it is important to notice another anomaly in the application of the Voigt transformations. If we apply the Lorentz transformations to (3) we obtain a set of expressions that are symmetric to (6) and reproduce exactly the expressions from the Einstein paper (A. Einstein., 1905):

$$\omega' = \omega\gamma(1-\frac{lv}{c})$$
$$l' = \frac{l-\frac{v}{c}}{1-\frac{lv}{c}}$$
$$m' = \frac{m}{\gamma(1-\frac{lv}{c})} \qquad (9)$$
$$n' = \frac{n}{\gamma(1-\frac{lv}{c})}$$

That is not the case if we apply the Voigt transformations, simple algebra produces:

$$\omega' = \omega\gamma^2(1-\frac{lv}{c})$$
$$l' = \frac{l-\frac{v}{c}}{1-\frac{lv}{c}}$$
$$m' = \frac{m}{\gamma(1-\frac{lv}{c})} \qquad (10)$$
$$n' = \frac{n}{\gamma(1-\frac{lv}{c})}$$

Once again, the expressions for aberration are correct while the expression for Doppler effect is not, resulting into an easy experimental falsification via the Ives-Stilwell class of experiments.

Before we conclude this section, it is worthwhile to note that other authors have noticed important shortcomings of the Voigt transformations. For example, Heras (R. Heras., n.d.) and Gluckman (A. G. Gluckman., 1968) have noticed that the Voigt transformations, unlike the Lorentz transformations, do not form a group. Gluckman (A. G. Gluckman., 1968) has also noticed that there is no way to arrive to the expression of relativistic total energy from the Voigt transformations.

4. A Voice in Favor

Ernst and Hsu (Ernst, J. Hsu., (2001) claim that Voigt transformations are fully equivalent with the Lorenz transformations. They start with the strange claim that, in reality the Voigt transformations are not between to arbitrary inertial frames but between an inertial frame and a preferred ("aether") frame:

$$x' = x - V_a t$$
$$y' = y/\gamma(V_a)$$
$$z' = z/\gamma(V_a) \qquad (11)$$
$$t' = t - \frac{V_a x}{c^2}$$

In (11) V_a is the speed between the inertial frame $F'(V_a)$ and the "aether" frame. This is very curious since nowhere in the original

German version of the Voigt paper there any mention of V_a or of a preferred frame or of any "aether". The authors proceed to derive the Doppler effect as:

$$\omega' = \omega_0' \sqrt{\frac{1+V_a/c}{1-V_a/c}}, \omega_0' = \omega' \,|_{\text{at rest in } F'(V_a)} \quad (12)$$

The authors seem completely oblivious that the formula depends on the unmeasurable speed V_a with respect to the impossible to determine "preferred" frame. We can track this error all the way back to their claim (16) that the Voigt transformations form a special case of the Poincare group with $\kappa = \sqrt{1-V_a^2/c^2}$. In the Poincare formulation, κ is a constant, whereas $\sqrt{1-V_a^2/c^2}$ clearly is a variable function of the speed of each inertial frame involved in the transformation with respect to the "preferred" frame.

5. Conclusion

We have examined different arguments pro and con the equivalence between the Lorentz transformations and the Voigt transformations. We have debunked some modern pro arguments and we have uncovered some novel con arguments in the contradictory fact that while the Voigt transformations produce the relativistic aberration expressions, they fail to produce, under any circumstances, the correct relativistic Doppler effect expressions. Therefore, we conclude that the Voigt transformations are not equivalent to the Lorentz transformations.

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