Comprehensive Benefit Evaluation of Intersections Controlled by Signals Based on Stochastic Frontiers

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doi:10.56397/JPEPS.2023.06.11

Abstract
As an important part of urban road traffic system, signal-controlled intersection has become one of the hot spots in urban road traffic research due to its geographical location and complex operation state. Due to the current goals and means of the optimal design of intersections are not clear enough, the intersection has not played a full role, and eventually the traffic efficiency is not high, congestion not only does not decrease but increases, but also leads to a high proportion of traffic accidents, hindering social economic development. In order to establish an evaluation system for signal-controlled intersections, this paper proposes a comprehensive evaluation index of signal-controlled intersections with traffic efficiency, safety and environment, and establishes an evaluation method for signal-controlled intersections based on stochastic frontier analysis. Finally, an example is used to verify that the method is feasible.

Keywords: signalized intersection, comprehensive benefit evaluation, traffic efficiency, traffic conflict, stochastic frontier

1. Introduction
With the continuous improvement of the market economy system, the economy has developed rapidly, the logistics industry and the travel rate of individuals have increased rapidly, so that the traffic flow on the road is also increasing, so it is imperative to accelerate the construction of road infrastructure. Under the premise of ensuring traffic safety, how to give full play to the function of the transportation hub of the intersection is also an important issue. The intersection controlled by the signal light is a very important node in urban road traffic, and it is also a place where traffic accidents often occur. The traffic efficiency of urban signal intersections in China is low, and traffic accidents are frequent, resulting in traffic congestion on surrounding roads, which brings many inconveniences to citizens' lives and travel (HU Yawen, 2016). Therefore, improving the efficiency, safety and environment of signaled intersections is critical to reducing the pressure on urban traffic (ZHAO Xing, LIN Hao, LIU Yi, et al., 2020). To comprehensively evaluate the performance, safety and environment of signal intersections, it is necessary to deeply analyze the problems existing in signal intersections and
put forward suggestions and countermeasures to effectively improve their traffic performance. The research on signal control intersection in foreign developed countries was carried out relatively early, and its theoretical system was more mature than that of domestic pairs. For example: Kim (KIM Y, CHOI S, YEO H., 2020; KIM Y, CHOI S, PARK J, et al., 2019) et al. proposed a traffic simulation model based on agent-based mesoscopic cell transport model (AMCTM). ZHENG (ZHENG X Y, DOU R, DING H, et al., 2011) through the in-depth analysis of the existing traffic status evaluation indicators, the recommended value of signal intersection service level corresponding to the comprehensive evaluation function is given. PATEL (PATEL N, MUKHE R JEE A B., 2015) uses traffic congestion values (TCVs) to quantitatively analyze each index. Wu Renliang, Zhou Jiajun, Zhou Dewenjie, Zhou Jiajun, Zhou De used stochastic frontier production function model to analyze the difference in port operation efficiency in China. Wan Li, Cheng Huiping, Cheng Yuqing (WAN Li, CHENG Huiping, CHENG Yuqing, 2017) selected stochastic frontier analysis, combined with information entropy method to measure and analyze the knowledge exchange efficiency of academic journals, and concluded that the key indicator to measure the influence of journals is knowledge exchange efficiency.

Based on the existing research, this paper uses the AHP method to quantitatively analyze the influencing factors of signal control intersection on the basis of selecting the main influencing factors of signal control intersection traffic efficiency, safety and environment, constructs an evaluation index system, and uses the stochastic frontier analysis method to evaluate the comprehensive benefit of signal control intersection. Several intersections in Jinzhong City are selected as examples for case verification.

2. Method

The intersection efficiency is a multi-input and multi-output production system. It is different from the characteristics of multi-input and single-output indicators in the SFA method. It emphasizes the comprehensive effect of multiple indicators. Therefore, when selecting output indicators, all influencing factors should be considered comprehensively to ensure the accuracy of output results. To this end, this paper uses the AHP method to weight the output index, and fits the multi-output index, and finally obtains a comprehensive output index (Zhang Le, 2017).

2.1 AHP Model

(1) Using expert scoring to construct a judgment matrix

$$A = \begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{n1} & a_{n2} & \cdots & a_{nn}
\end{bmatrix}_{m \times n} \quad (1)$$

Where $a_{ij}$ is the degree of importance between the two indicators, the specific score is shown in table 1.

Table 1. Scale meaning

<table>
<thead>
<tr>
<th>scale</th>
<th>definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Just as important</td>
</tr>
</tbody>
</table>
One object is slightly more important than the other.
One object is more important than the other.
One object is quite more important than the other.
One object is largely more important than the other.

Comparison conclusions: Between the above analysis values, the results are in the median of the above grade.

If factor $i$ is compared with factor $j$ to get the score $r_{ij}$, then factor $j$ is compared with factor $i$ to get the reciprocal of $r_{ij}$.

(2) Hierarchical single sort calculation
The column vector of matrix $A$ is normalized to obtain a normalized judgment matrix, and then the arithmetic mean value of each row in the matrix is calculated. That is, the eigenvector $W = (w_1, w_2, w_3, \ldots, w_n)$ of matrix $A$ can be obtained. The formula of $\lambda_{max}$ is as follows:

$$\lambda_{max} = \sum_{i=1}^{n} \frac{(AW)_i}{nW_i} = \frac{1}{n} \sum_{j=1}^{n} \frac{a_{ij}W_j}{W_j}$$

In the formula, $\lambda_{max}$ is the maximum eigenvalue of the judgment matrix; $n$ is the number of rows of the judgment matrix; $W$ is the eigenvector of the judgment matrix; $(AW)_i$ is the $i$th element of the vector $AW$ obtained by multiplying the judgment matrix $A$ and the eigenvector $W$.

(3) Consistency test
In order to test and evaluate the consistency of expert thinking consciousness, $CI$ is introduced to measure the deviation of judgment matrix. The calculation formula is:

$$CI = \frac{\lambda_{max} - n}{n - 1}$$

The random consistency ratio $CR$ is calculated as:

$$CR = \frac{CI}{RI}$$

Among them, the $RI$ value can be determined according to the values given in the table. See Table 2. When $CR \leq 0.10$, it indicates that the judgment matrix has good consistency and the result is relatively accurate. Otherwise, the consistency test will have defects and need to be modified and adjusted.

<table>
<thead>
<tr>
<th>$n$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
</table>
| $RI$ | 0  | 0  | 0.515 | 0.893 | 1.119 | 1.249 |}

2.2 SFA model
2.2.1 Composition of the Model
SFA uses the maximum likelihood estimation method to estimate the parameters, and then combines the conditional expectation value, and finally obtains the technical efficiency of each decision-making unit, so as to realize the ‘equal’ treatment of each decision-making unit. The specific steps are as follows:

$$\ln y_{it} = \ln f(x_{it}, \beta) + v_{it} - \mu_{it} (i = 1, 2, \ldots, n; t = 1, 2, \ldots, T)$$

$$\mu_{it} = \delta(t)\mu_i, \delta(t) = \exp\{\eta(T-t)\}$$

$$TE_{it} = E(\exp\{-\mu_{it}\} | \varepsilon_{it})$$

$$\gamma = \frac{\sigma^2_u}{\sigma^2_u + \sigma^2_v}$$

In the formula: $\sigma^2_u$, $\sigma^2_v$, $\eta$, $\gamma$, and $\beta$ are parameters to be estimated. In the above model, $y_{it}$ is the actual output of the $i$th decision unit in the $t$th period; $x_{it}$ is the cost input of the $i$th decision making unit in the $t$ period; $f(x_{it}, \beta)$ is the selected production function; the random influence factor is represented by $v_{it}$, because this random factor may lead to the increase or decrease of production, so it is often assumed that $v_{it}$ is i.i.d $N(0, \sigma^2)$.  

2.2.2 Production Function Selection
Production function is a mathematical expression of the quantitative relationship between factor input and output involved in production activities. He expressed the relationship between the proportion of production factors in the production process.
and the maximum output in a certain period of time when a certain technical level remains unchanged.

The production function of SFA is generally C-D function and Translog function. If only the two inputs of capital (K) and labor (L) are considered, it can be known that the C-D function can be expressed as the following linear form after taking the natural logarithm (George E. Battese & Sumiter S. Broca., 1997):

$$\ln y = \beta_0 + \beta_1 \ln K + \beta_2 \ln L$$

(10)

Where $\beta_0$, $\beta_1$, and $\beta_2$ are the parameters to be estimated.

As an extension of the C-D function, the Translog function has the following form:

$$\ln y = \beta_0 + \beta_1 \ln K + \beta_2 \ln L + \beta_3 (\ln K)^2 + \beta_4 (\ln L)^2 + \beta_5 \ln K \ln L$$

(11)

Where $\beta_0$, $\beta_1$, $\beta_2$, $\beta_3$, $\beta_4$, and $\beta_5$ are the parameters to be estimated.

In this paper, the stochastic frontier model of C-D production function in logarithmic form is adopted. The calculation is shown as follows:

$$\ln y_{it} = \beta_0 + \beta_1 \ln x_{it} + v_{it} + \mu_{it}$$

(12)

$$TE_{it} = e^{-\mu_{it}}$$

(13)

$$\gamma = \frac{\sigma_v^2}{\sigma^2 + \sigma^2 \mu}$$

(14)

In the formula: $i$ represents the order of intersections; $t$ represents the number of periods; $y_{it}$ represents the output variable of intersection traffic benefit; $\beta_0$ is the intercept term; $\beta_1$ is the parameter to be estimated, which represents the output elasticity of the single-lane design capacity of the intersection, the number of vehicles arriving at the intersection, the number of entrance lanes at the intersection, the number of exit lanes at the intersection, the signal cycle of the intersection, and the effective green time of the intersection. $v_{it}$ is a random variable obeying normal distribution $N(0, \sigma^2)$; $\mu_{it}$ is a technical inefficiency term, which is assumed to follow a truncated normal distribution. $\gamma$ is used to test the applicability of the evaluation model. When its value is closer to 1, it means that the model is more suitable for stochastic frontier analysis.

3. Indicator System

(1) Queue length at intersection

The queue length is visible to the naked eye and easy to observe, which is the only choice as the evaluation index of signal control intersection. This paper selects the maximum queue length as the evaluation index.

(2) Capacity of signalized intersection

There are many factors affecting the capacity of the intersection, such as the shape and size of the intersection, the number of lanes at the intersection, the width of the intersection, the traffic command and traffic facilities at the intersection, and the time of the signal lights at the intersection. These factors will directly affect the capacity of the signalized intersection.

(3) Intersection delay

Traffic delay refers to the vehicle driving on a road for a longer time than normal running time. It does not mean that there will be delays on all roads.

(4) Saturation

Saturation has a great influence on traffic flow. When the traffic flow at the intersection reaches a certain level, it will increase the saturation of the intersection and increase the residence time of the vehicle at the intersection, which will lead to more delays at the intersection, thereby reducing the traffic flow at the intersection.

(5) Traffic conflict

There are many factors that affect the safety of intersections. The core problem is the potential safety hazards caused by traffic conflicts. The conflict points are opposite to the safety of intersections. The fewer the points, the higher the safety. Otherwise, the security is lower.

(6) Geometric characteristics of intersection

The geometric characteristics of intersections are the root causes of weaving and conflicts, including lane setting, intersection sight distance, and intersection angle (Zhang Le, 2017). These data need to be strictly measured and analyzed to ensure their accuracy. In the process of drawing on relevant norms and standards, we must first analyze the actual survey and inquiry to obtain accurate data.

(7) Pollutant emissions

Taking the emissions of CO, NOx and HC in the vehicle exhaust at the intersection as the
evaluation index, the total emissions of CO, NOx and HC in the vehicle exhaust were estimated by the specific power method (SHI Jihong, 2012).

(8) Noise

In this paper, the equivalent A sound level is used to represent the noise pollution of signalized intersections.

4. Case Study

The location of this paper is Zhongdu Road-Yingbin Street, Zhongdu Road-Wenyuan Street, Xinjian Road-Yingbin Street, Xinjian Road-Wenyuan Street intersection. The four intersections selected are the main road intersections in the urban center, with large traffic flow and frequent traffic congestion. It has practical reference value for its benefit evaluation.

The basic geometric characteristics of the intersection are shown in Figure 1, Figure 2, Figure 3 and Figure 4. The results of the intersection survey data are shown in Table 3.

<table>
<thead>
<tr>
<th>road junction</th>
<th>peak hour volume</th>
<th>Average capacity (pcu/h)</th>
<th>degree of saturation (V/C)</th>
<th>average queue length</th>
<th>vehicle delay (s)</th>
<th>average running speed</th>
<th>Traffic conflict level</th>
<th>Geometric feature score</th>
</tr>
</thead>
</table>

Figure 1. Schematic map of the current intersection of Zhongdu Road and Yingbin Street

Figure 2. Schematic map of the current intersection of Zhongdu Road and Wenyuan Street

Figure 3. Schematic map of the current intersection of Xinjian Road - Yingbin Street

Figure 4. Schematic map of the current intersection of Xinjian Road - Wenyuan Street

Table 3. Intersection survey data
According to the scores of experts, the weights of each scheme layer and criterion layer to the target layer are calculated as Table 4.

<table>
<thead>
<tr>
<th></th>
<th>Weight</th>
<th>Weight</th>
<th>Weight</th>
<th>Weight</th>
<th>Weight</th>
<th>Weight</th>
<th>Weight</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traffic benefit of intersection</td>
<td>0.5126</td>
<td>0.3904</td>
<td>0.0970</td>
<td>0.7023</td>
<td>0.2977</td>
<td>0.7789</td>
<td>0.2211</td>
<td></td>
</tr>
<tr>
<td>Traffic efficiency</td>
<td>0.5126</td>
<td>0.3904</td>
<td>0.0970</td>
<td>0.7023</td>
<td>0.2977</td>
<td>0.7789</td>
<td>0.2211</td>
<td></td>
</tr>
<tr>
<td>Safety</td>
<td>0.3904</td>
<td>0.2977</td>
<td>0.0970</td>
<td>0.7789</td>
<td>0.2211</td>
<td>0.2904</td>
<td>0.1707</td>
<td></td>
</tr>
<tr>
<td>Environment</td>
<td>0.0970</td>
<td>0.2211</td>
<td>0.2977</td>
<td>0.7789</td>
<td>0.0970</td>
<td>0.2904</td>
<td>0.1707</td>
<td></td>
</tr>
</tbody>
</table>

The weights of the three output indicators of traffic efficiency, safety and environment are 0.5126, 0.3904 and 0.0970 respectively. It can be seen that traffic efficiency can better reflect the benefits of intersections, and relying solely on traffic efficiency is not enough to fully measure the comprehensive benefits of intersections. Therefore, it is necessary to calculate the comprehensive output index.

The STATA software is used to test and estimate the traffic data of the intersection of Zhongdu Road-Yingbin Street, Zhongdu Road-Wenyuan Street, Xinjian Road-Yingbin Street and Xinjian Road-Wenyuan Street for one week. The estimated results of the intersection traffic benefit production function model are shown in Figure 5.
Traffic volume as the maximum output elasticity has the greatest impact on the intersection benefit. The effective green time has the least impact on the intersection benefit. The length of signal cycle is negatively correlated with traffic benefit and plays a negative role in traffic benefit. The number of lanes is positively correlated with traffic benefits, indicating that increasing the number of lanes has a good effect on traffic benefits.

The efficiency value and change of each intersection can be seen from figure 6. The overall efficiency remains stable, and the order of intersection benefits from high to low is: Zhongdu Road-Yingbin Street, Xinjian Road-Wenyuan Street, Zhongdu Road-Wenyuan Street, Zhongdu Road-Yingbin Street. In this regard, we can conclude that the priority is to optimize the intersection of Zhongdu Road-Yingbin Street.

In order to verify the rationality of the evaluation method, VISSSIM is used to simulate and compare the initial scheme and the optimization scheme, and the vehicle operation status of several intersections in Jinzhong City is analyzed by expert scoring method. The evaluation results of the expert scoring method are compared with the evaluation results of the stochastic frontier analysis, as shown in Table 5. The evaluation results are basically the same, indicating that the results of stochastic frontier analysis are effective.

<table>
<thead>
<tr>
<th>Before optimization</th>
<th>SFA</th>
<th>expert scoring</th>
<th>efficiency sequencing</th>
<th>SFA</th>
<th>expert scoring</th>
<th>efficiency sequencing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zhongdu Road-Yingbin Street</td>
<td>0.62</td>
<td>64</td>
<td>4</td>
<td>0.67</td>
<td>80</td>
<td>2</td>
</tr>
<tr>
<td>Chungdu Road-Wenyuan Street</td>
<td>0.64</td>
<td>74</td>
<td>3</td>
<td>0.64</td>
<td>74</td>
<td>4</td>
</tr>
<tr>
<td>New Road-Yingbin Street</td>
<td>0.68</td>
<td>82</td>
<td>1</td>
<td>0.68</td>
<td>82</td>
<td>1</td>
</tr>
<tr>
<td>New Road-Wenyuan Street</td>
<td>0.65</td>
<td>78</td>
<td>2</td>
<td>0.65</td>
<td>78</td>
<td>3</td>
</tr>
</tbody>
</table>

5. Conclusion

Based on the existing research on intersection benefit evaluation, this paper selects traffic...
efficiency, safety and environment as output indicators, and constructs an evaluation index system. Through case analysis, four intersections in Jinchong City are selected as research objects, and their indicators are analyzed. An evaluation method of signal-controlled intersections based on stochastic frontier analysis model is established. Through stochastic frontier analysis, the existing problems of urban intersections can be accurately identified. It can not only determine the intersections that need to be optimized, but also clarify the direction of optimization, and has practical guidance for the transformation of urban signal-controlled intersections.

Due to the limited research level and the technology, data and other reasons, the evaluation index system of signalized intersection has a certain subjective tendency. In the follow-up study, different methods can be combined to reduce the result error caused by subjectivity, ensure the objectivity of the results and improve the accuracy of the results.

**Fund Project**

This research was supported by Graduate Education Innovation Project of Taiyuan University of Science and Technology (SY2022065); Shanxi Postgraduate Education Reform Project (2021YJG249)

**References**


