

# Comprehensive Benefit Evaluation of Intersections Controlled by Signals Based on Stochastic Frontiers

Shaochen Feng<sup>1</sup> & Zhixuan Jia<sup>1</sup>

<sup>1</sup> School of Vehicle and Traffic Engineering, Taiyuan University of Science and Technology, Taiyuan 030024, China

Correspondence: Zhixuan Jia, School of Vehicle and Traffic Engineering, Taiyuan University of Science and Technology, Taiyuan 030024, China.

doi:10.56397/JPEPS.2023.06.11

## Abstract

As an important part of urban road traffic system, signal-controlled intersection has become one of the hot spots in urban road traffic research due to its geographical location and complex operation state. Due to the current goals and means of the optimal design of intersections are not clear enough, the intersection has not played a full role, and eventually the traffic efficiency is not high, congestion not only does not decrease but increases, but also leads to a high proportion of traffic accidents, hindering social economic development. In order to establish an evaluation system for signal-controlled intersections, this paper proposes a comprehensive evaluation index of signal-controlled intersections with traffic efficiency, safety and environment, and establishes an evaluation method for signal-controlled intersections based on stochastic frontier analysis. Finally, an example is used to verify that the method is feasible.

**Keywords:** signalized intersection, comprehensive benefit evaluation, traffic efficiency, traffic conflict, stochastic frontier

## 1. Introduction

With the continuous improvement of the market economy system, the economy has developed rapidly, the logistics industry and the travel rate of individuals have increased rapidly, so that the traffic flow on the road is also increasing, so it is imperative to accelerate the construction of road infrastructure. Under the premise of ensuring traffic safety, how to give full play to the function of the transportation hub of the intersection is also an important issue. The intersection controlled by the signal light is a very important node in urban road traffic, and it

is also a place where traffic accidents often occur. The traffic efficiency of urban signal intersections in China is low, and traffic accidents are frequent, resulting in traffic congestion on surrounding roads, which brings many inconveniences to citizens' lives and travel (HU Yawen, 2016). Therefore, improving the efficiency, safety and environment of signaled intersections is critical to reducing the pressure on urban traffic (ZHAO Xing, LIN Hao, LIU Yi, et al., 2020). To comprehensively evaluate the performance, safety and environment of signal intersections, it is necessary to deeply analyze the problems existing in signal intersections and

put forward suggestions and countermeasures to effectively improve their traffic performance.

The research on signal control intersection in foreign developed countries was carried out relatively early, and its theoretical system was more mature than that of domestic pairs. For example: Kim (KIM Y, CHOI S, YEO H., 2020; KIM Y, CHOI S, PARK J, et al., 2019) et al. proposed a traffic simulation model based on agent-based mesoscopic cell transport model (AMCTM). ZHENG (ZHENG X Y, DOU R, DING H, et al., 2011) through the in-depth analysis of the existing traffic status evaluation indicators, the recommended value of signal intersection service level corresponding to the comprehensive evaluation function is given. PATEL (PATEL N, MUKHE R JEE A B., 2015) uses traffic congestion values (TCVs) to quantify traffic congestion in different spatial areas of different land-use types.

In China, research on signal-controlled intersection evaluation has also been carried out one after another. Shi Jihong (SHI Jihong, 2012) took urban road intersections as the research object to study the dispersion law of motor vehicle pollutants in urban roads, the hourly average emission of pollutants in urban road traffic environment, and the overall traffic environment quality assessment of urban road environment. Zeng Rong, Zhang Cunbao, Cao Yu (ZENG Rong, ZHANG Cunbao, CAO Yu, et al., 2022) et al. according to the characteristics of track data, extracted vehicle conflict data, and replaced accident data with traffic conflict technology for intersection safety evaluation. Wu Renliang (WU Renliang, YANG Hao, LIU Tong, 2019) et al. took peak and flat peak periods as the evaluation objects, analyzed various existing evaluation indicators and evaluation methods, calculated the intersection signal control benefit index by weighted average, and quantitatively analyzed each index.

Stochastic Frontier Analysis (SFA) is widely used in technical efficiency measurement. He Wenjie, Zhou Jiajun, Zhou De (HE Wenjie, ZHOU Jiajun & ZHOU De, 2021) used the stochastic frontier production function model to compare and analyze the technical efficiency of pear production in different regions of China. Ju Shuimu (Ju Shuimu, 2019) used panel data to analyze the difference in port operation efficiency in China. Wan Li, Cheng Huiping, Cheng Yuqing (WAN Li, CHENG Huiping, CHENG Yuqing, 2017) selected stochastic

frontier analysis, combined with information entropy method to measure and analyze the knowledge exchange efficiency of academic journals, and concluded that the key indicator to measure the influence of journals is knowledge exchange efficiency.

Based on the existing research, this paper uses the AHP method to quantitatively analyze the influencing factors of signal control intersection on the basis of selecting the main influencing factors of signal control intersection traffic efficiency, safety and environment, constructs an evaluation index system, and uses the stochastic frontier analysis method to evaluate the comprehensive benefit of signal control intersection. Several intersections in Jinzhong City are selected as examples for case verification.

## 2. Method

The intersection efficiency is a multi-input and multi-output production system. It is different from the characteristics of multi-input and single-output indicators in the SFA method. It emphasizes the comprehensive effect of multiple indicators. Therefore, when selecting output indicators, all influencing factors should be considered comprehensively to ensure the accuracy of output results. To this end, this paper uses the AHP method to weight the output index, and fits the multi-output index, and finally obtains a comprehensive output index (Zhang Le, 2017).

### 2.1 AHP Model

(1) Using expert scoring to construct a judgment matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}_{n \times n} \quad (1)$$

Where  $a_{11}$  is the degree of importance between the two indicators, the specific score is shown in table 1.

**Table 1.** Scale meaning

scale	definition
1	Just as important

3	One object is slightly more important than the other.
5	One object is more important than the other.
7	One object is quite more important than the other.
9	One object is largely more important than the other.
2, 4, 6, 8	Comparison conclusion Between the above analysis values, the results are in the median of the above grade.
fraction	If factor $i$ is compared with factor $j$ to get the score $r_{ij}$ , then factor $j$ is compared with factor $i$ to get the reciprocal of $r_{ij}$ .

## (2) Hierarchical single sort calculation

The column vector of matrix A is normalized to obtain a normalized judgment matrix, and then the arithmetic mean value of each row in the matrix is calculated. That is, the eigenvector  $W=(w_1, w_2, w_3, \dots, w_n)$  of matrix A can be obtained. The formula of  $\lambda_{max}$  is as follows:

$$\lambda_{\max} = \sum_{i=1}^n \frac{(AW)_i}{nW_i} = \frac{1}{n} \sum_{j=1}^n \frac{\sum_{i=1}^n a_{ij}W_j}{W_j} \quad (2)$$

In the formula:  $\lambda_{max}$  is the maximum eigenvalue of the judgment matrix;  $n$  is the number of rows of the judgment matrix;  $W$  is the eigenvector of the judgment matrix;  $(AW)_i$  is the  $i$  th element of the vector  $AW$  obtained by multiplying the judgment matrix A and the eigenvector  $W$ .

## (3) Consistency test

In order to test and evaluate the consistency of expert thinking consciousness,  $CI$  is introduced to measure the deviation of judgment matrix. The calculation formula is:

$$CI = \frac{\lambda_{\max} - n}{n - 1} \quad (3)$$

The random consistency ratio  $CR$  is calculated as:

$$CR = \frac{CI}{RI} \quad (4)$$

Among them, the  $RI$  value can be determined

according to the values given in the table. See Table 2. When  $CR \leq 0.10$ , it indicates that the judgment matrix has good consistency and the result is relatively accurate. Otherwise, the consistency test will have defects and need to be modified and adjusted.

**Table 2.** Summary table of consistency metric values

$n$	1	2	3	4	5	6
$RI$	0	0	0.515	0.893	1.119	1.249

## 2.2 SFA model

### 2.2.1 Composition of the Model

SFA uses the maximum likelihood estimation method to estimate the parameters, and then combines the conditional expectation value, and finally obtains the technical efficiency of each decision-making unit, so as to realize the 'equal' treatment of each decision-making unit. The specific steps are as follows:

$$\ln y_{it} = \ln f(x_{it}, \beta) + v_{it} - \mu_{it} \quad (5)$$

( $i = 1, 2, \dots, n; t = 1, 2, \dots, T$ )

$$\mu_{it} = \delta(t)\mu_i, \delta(t) = \exp\{\eta(T-t)\} \quad (6)$$

$$TE_{it} = E\left(\exp\{-\mu_{it}\} \mid \varepsilon_{it}\right) \quad (7)$$

$$\gamma = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_v^2} \quad (8)$$

In the formula :  $\sigma_v^2$ 、 $\sigma_u^2$ 、 $\eta$ 、 $\gamma$ 、and  $\beta$  are parameters to be estimated. In the above model,  $y_{it}$  is the actual output of the  $i$ th decision unit in the  $t$ th period;  $x_{it}$  is the cost input of the  $i$  th decision making unit in the  $t$  period;  $f(x_{it}, \beta)$  is the selected production function; the random influence factor is represented by  $v_{it}$ , because this random factor may lead to the increase or decrease of production, so it is often assumed that  $v_{it} \sim i.i.dN(0, \sigma_v^2)$ .

### 2.2.2 Production Function Selection

Production function is a mathematical expression of the quantitative relationship between factor input and output involved in production activities. He expressed the relationship between the proportion of production factors in the production process

and the maximum output in a certain period of time when a certain technical level remains unchanged.

The production function of SFA is generally C-D function and Translog function. If only the two inputs of capital (K) and labor (L) are considered, it can be known that the C-D function can be expressed as the following linear form after taking the natural logarithm (George E. Battese & Sumiter S. Broca., 1997):

$$Ln f = \beta_0 + \beta_1 \ln K + \beta_2 \ln L \quad (10)$$

Where  $\beta_0$ ,  $\beta_1$  and  $\beta_2$  are the parameters to be estimated.

As an extension of the C-D function, the Translog function has the following form:

$$Ln f = \beta_0 + \beta_1 \ln K + \beta_2 \ln L + \beta_3 (\ln K)^2 + \beta_4 (\ln L)^2 + \beta_5 \ln K \ln L \quad (11)$$

Where  $\beta_0$ ,  $\beta_1$ ,  $\beta_2$ ,  $\beta_3$ ,  $\beta_4$  and  $\beta_5$  are the parameters to be estimated.

In this paper, the stochastic frontier model of C-D production function in logarithmic form is adopted. The calculation is shown as follows:

$$\ln y_{it} = \beta_0 + \beta_i \ln x_{it} + v_{it} + \mu_{it} \quad (12)$$

$$TE_{it} = e^{-\mu_{it}} \quad (13)$$

$$\gamma = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\mu^2} \quad (14)$$

In the formula:  $i$  represents the order of intersections,  $t$  represents the number of periods;  $y_{it}$  represents the output variable of intersection traffic benefit;  $\beta_0$  is the intercept term;  $\beta_i$  is the parameter to be estimated, which represents the output elasticity of the single-lane design capacity of the intersection, the number of vehicles arriving at the intersection, the number of entrance lanes at the intersection, the number of exit lanes at the intersection, the signal cycle of the intersection, and the effective green time of the intersection.  $v_{it}$  is a random variable obeying normal distribution  $N(0, \sigma_v^2)$ ;  $\mu_{it}$  is a technical inefficiency term, which is assumed to follow a truncated normal distribution.  $\gamma$  is used to test the applicability of the evaluation model. When its value is closer to 1, it means that the model is more suitable for stochastic frontier

analysis.

### 3. Indicator System

#### (1) Queue length at intersection

The queue length is visible to the naked eye and easy to observe, which is the only choice as the evaluation index of signal control intersection. This paper selects the maximum queue length as the evaluation index.

#### (2) Capacity of signalized intersection

There are many factors affecting the capacity of the intersection, such as the shape and size of the intersection, the number of lanes at the intersection, the width of the intersection, the traffic command and traffic facilities at the intersection, and the time of the signal lights at the intersection. These factors will directly affect the capacity of the signalized intersection.

#### (3) Intersection delay

Traffic delay refers to the vehicle driving on a road for a longer time than normal running time. It does not mean that there will be delays on all roads.

#### (4) Saturation

Saturation has a great influence on traffic flow. When the traffic flow at the intersection reaches a certain level, it will increase the saturation of the intersection and increase the residence time of the vehicle at the intersection, which will lead to more delays at the intersection, thereby reducing the traffic flow at the intersection.

#### (5) Traffic conflict

There are many factors that affect the safety of intersections. The core problem is the potential safety hazards caused by traffic conflicts. The conflict points are opposite to the safety of intersections. The fewer the points, the higher the safety. Otherwise, the security is lower.

#### (6) Geometric characteristics of intersection

The geometric characteristics of intersections are the root causes of weaving and conflicts, including lane setting, intersection sight distance, and intersection angle (Zhang Le, 2017). These data need to be strictly measured and analyzed to ensure their accuracy. In the process of drawing on relevant norms and standards, we must first analyze the actual survey and inquiry to obtain accurate data.

#### (7) Pollutant emissions

Taking the emissions of CO, NOx and HC in the vehicle exhaust at the intersection as the

evaluation index, the total emissions of CO, NO<sub>x</sub> and HC in the vehicle exhaust were estimated by the specific power method (SHI Jihong, 2012).

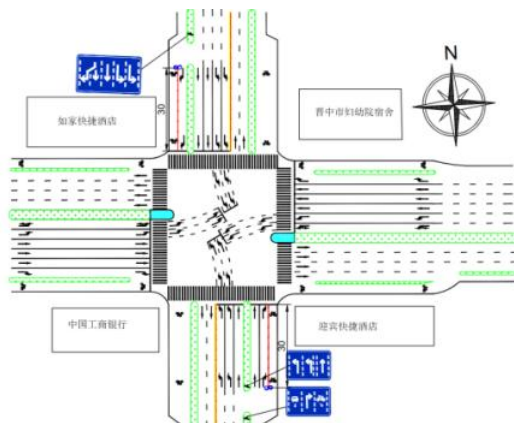
#### (8) Noise

In this paper, the equivalent A sound level is used to represent the noise pollution of signalized intersections.

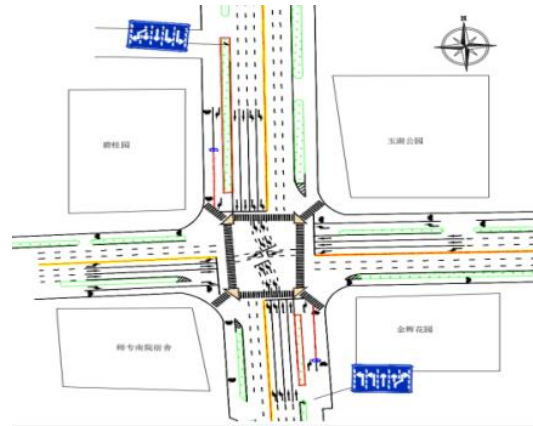
#### 4. Case Study

The location of this paper is Zhongdu Road-Yingbin Street, Zhongdu Road-Wenyuan Street, Xinjian Road-Yingbin Street, Xinjian Road-Wenyuan Street intersection. The four intersections selected are the main road intersections in the urban center, with large traffic flow and frequent traffic congestion. It has practical reference value for its benefit evaluation.

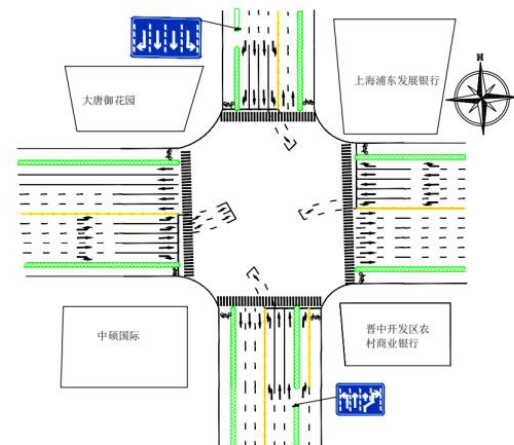
The basic geometric characteristics of the intersection are shown in Figure 1, Figure 2, Figure 3 and Figure 4. The results of the intersection survey data are shown in Table 3.



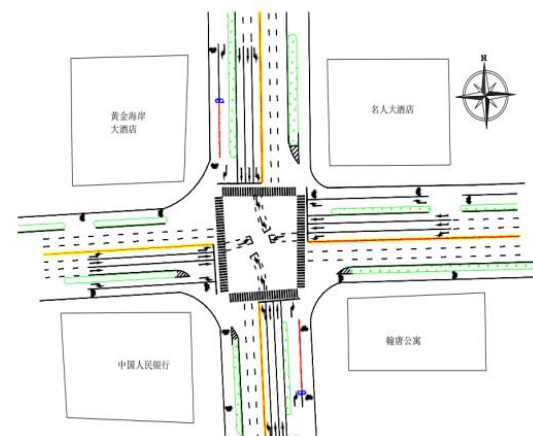
**Figure 1.** Schematic map of the current intersection of Zhongdu Road and Yingbin Street



**Figure 2.** Schematic map of the current intersection of Zhongdu Road and Wenyuan Street



**Figure 3.** Schematic map of the current intersection of Xinjian Road - Yingbin Street



**Figure 4.** Schematic map of the current intersection of Xinjian Road - Wenyuan Street

**Table 3.** Intersection survey data

road junction	peak hour volume	Average capacity (pcu/h)	degree of saturation (V/C)	average queue length	vehicle delay (s)	average running speed	Traffic conflict level	Geometric feature score
---------------	------------------	--------------------------	----------------------------	----------------------	-------------------	-----------------------	------------------------	-------------------------



	(pcu/h)			(m)		(km/h)		
Zhongdu Road-Yingbin Street	1532	685	0.75	38	45.3	29.6	12	82
Chungdu Road - Wenyuan Street	1568	692	0.73	36	43.6	31.5	13	83
New Road - Yingbin Street	1463	695	0.68	30	40.1	32.7	11	85
New Road-Wenyuan Street	1492	688	0.72	32	41.8	32.4	12	85

According to the scores of experts, the weights of each scheme layer and criterion layer to the target layer are calculated as Table 4.

**Table 4.** Evaluation metric weights

goal layer	criterion layer	weight	schematic layer	weight
Traffic benefit of intersection	efficiency	0.5126	queue length	0.2904
			traffic capacity	0.1707
			vehicle delay	0.2497
			degree of saturation	0.1824
			vehicle speed	0.1068
	safety	0.3904	traffic conflict	0.7023
			Geometric characteristics of intersection	0.2977
	environment	0.0970	pollutant emission	0.7789
			noise	0.2211

The weights of the three output indicators of traffic efficiency, safety and environment are 0.5126, 0.3904 and 0.0970 respectively. It can be seen that traffic efficiency can better reflect the benefits of intersections, and relying solely on traffic efficiency is not enough to fully measure the comprehensive benefits of intersections. Therefore, it is necessary to calculate the comprehensive output index.

The STATA software is used to test and estimate the traffic data of the intersection of Zhongdu Road-Yingbin Street, Zhongdu Road-Wenyuan Street, Xinjian Road-Yingbin Street and Xinjian Road-Wenyuan Street for one week. The

estimated results of the intersection traffic benefit production function model are shown in Figure 5.

Log likelihood = 6.6954753					
Obs per group:					
				min =	7
				avg =	7
				max =	7
				Wald chi2(6) =	491.36
				Prob > chi2 =	0.0000
lny	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
lnx1	21.87959	13.81313	1.58	0.11	-5.193642 48.95283
lnx2	8.234149	6.963151	1.18	0.23	-5.413377 21.88168
lnx3	3.938377	1.018664	3.87	0.000	1.941832 5.934923
lnx4	4.500307	.6140717	2.44	0.015	.2967485 2.703865
lnx5	-5.680319	4.100819	-1.87	0.061	-15.71778 .3571374
lnx6	4.798493	2.455515	1.95	0.051	-.0142287 9.611215
_cons	-221.2056	120.9892	-1.83	0.068	-458.3401 15.92881
/mu	.0836402	.5071634	0.16	0.869	-.9103817 1.077662
/lnsigma2	-3.311348	.2805186	-11.80	0.000	-3.861154 -2.761542
/lgtgamma	-3.336812	6.358539	-0.52	0.600	-15.79932 9.125697
sigma2	.231514	1.166011			.121736 4483.523
gamma	.8461459	2.297331			1.55e-08 2
sigma_u2	.1264404	1.168196			-2.163182 2.416061
sigma_v2	.1050736	.0306547			.0449914 .1651556

**Figure 5.** Regression results of STATA

The efficiency values of each intersection are calculated by the efficiencies function, and the calculation results are shown in Figure 6.

```
> efficiencies(result)
```

	1	2	3	4	5	6	7
1	0.6254223	0.6139547	0.6253444	0.6293642	0.6238321	0.6281670	0.6109876
2	0.6423920	0.6452818	0.6472936	0.6418003	0.6497126	0.6462173	0.6415035
3	0.6823025	0.6777323	0.6791325	0.6833492	0.6800125	0.6693306	0.6936239
4	0.6528997	0.6671623	0.6495873	0.6549355	0.6544980	0.6600545	0.6522873

**Figure 6.** The efficiency of intersections

Gamma is the parameter of maximum likelihood estimation. It can be seen from Figure 6 that gamma = 0.846 approaches 1, indicating that most of the errors come from technical inefficiency, indicating that it is suitable for SEA model.

It can be seen from Figure 5 that the influence and significance level of each input variable in the production function model are different.

Traffic volume as the maximum output elasticity has the greatest impact on the intersection benefit. The effective green time has the least impact on the intersection benefit. The length of signal cycle is negatively correlated with traffic benefit and plays a negative role in traffic benefit. The number of lanes is positively correlated with traffic benefits, indicating that increasing the number of lanes has a good effect on traffic benefits.

The efficiency value and change of each intersection can be seen from figure 6. The overall efficiency remains stable, and the order of intersection benefits from high to low is: Xinjian Road-Yingbin Street, Xinjian Road-Wenyuan Street, Zhongdu Road-Wenyuan Street, Zhongdu Road-Yingbin Street. In this regard, we can conclude that the priority is to optimize the intersection of Zhongdu Road-Yingbin Street.

In order to verify the rationality of the evaluation method, VISSIM is used to simulate and compare the initial scheme and the optimization scheme, and the vehicle operation status of several intersections in Jinzhong City is analyzed by expert scoring method. The evaluation results of the expert scoring method are compared with the evaluation results of the stochastic frontier analysis, as shown in Table 5. The evaluation results are basically the same, indicating that the results of stochastic frontier analysis are effective.

**Table 5.** Compare the results of the analysis

road junction	Before optimization			optimized		
	SFA	expert scoring	efficiency sequencing	SFA	expert scoring	efficiency sequencing
Zhongdu Road-Yingbin Street	0.62	64	4	0.67	80	2
Chungdu Road - Wenyuan Street	0.64	74	3	0.64	74	4
New Road - Yingbin Street	0.68	82	1	0.68	82	1
New Road-Wenyuan Street	0.65	78	2	0.65	78	3

## 5. Conclusion

Based on the existing research on intersection benefit evaluation, this paper selects traffic

efficiency, safety and environment as output indicators, and constructs an evaluation index system. Through case analysis, four intersections in Jinzhong City are selected as research objects, and their indicators are analyzed. An evaluation method of signal-controlled intersections based on stochastic frontier analysis model is established. Through stochastic frontier analysis, the existing problems of urban intersections can be accurately identified. It can not only determine the intersections that need to be optimized, but also clarify the direction of optimization, and has practical guidance for the transformation of urban signal-controlled intersections.

Due to the limited research level and the technology, data and other reasons, the evaluation index system of signalized intersection has a certain subjective tendency. In the follow-up study, different methods can be combined to reduce the result error caused by subjectivity, ensure the objectivity of the results and improve the accuracy of the results.

### Fund Project

This research was supported by Graduate Education Innovation Project of Taiyuan University of Science and Technology (SY2022065); Shanxi Postgraduate Education Reform Project (2021YJJG249)

### References

George E. Battese, Sumiter S. Broca. (1997). Functional Forms of Stochastic Frontier Production Functions and Models for Technical Inefficiency Effects: A Comparative Study for Wheat Farmers in Pakistan. *Boston: Journal of Productivity Analysis*, (8), Kluwer Academic Publishers.

HE Wenjie, ZHOU Jiajun, ZHOU De. (2021). Stochastic frontier analysis of pear production technology efficiency in China. *Northern Fruits*, 223(03), 10-13. DOI:10.16376/j.cnki.bfsgs.2021.03.003.

HU Yawen. (2016). The main problems, causes and countermeasures of transportation in large cities. *Science and Technology*, 26(35), 225-226.

Ju Shuimu. (2019). Port operation efficiency evaluation based on panel stochastic frontier analysis. *Journal of Huzhou University*, 41(08), 111-116.

KIM Y, CHOI S, PARK J, et al. (2019). Agent-based mesoscopic urban traffic simulation based on multi-lane cell transmission model. *Procedia Computer Science*, 151, 240-247.

KIM Y, CHOI S, YEO H. (2020). Extended urban cell transmission model using agent-based modeling. *Procedia Computer Science*, 170, 354-361.

PATEL N, MUKHE R JEE A B. (2015). Assessment of Network Traffic Congestion through Traffic Congestability Value (TCV): A New Index. *Bulleting of Geography. Socioeconomic Series*, 30(30), 123-134.

SHI Jihong. (2012). Analysis and comprehensive evaluation of traffic pollution at urban road level intersections. Jilin University.

WAN Li, CHENG Huiping, CHENG Yuqing. (2017). Research on knowledge exchange efficiency of academic journals based on stochastic frontier analysis method. *Journal of Modern Information*, 37(06), 68-71+79.

WU Renliang, YANG Hao, LIU Tong. (2019). Evaluation index and method of road intersection signal control benefit. *Journal of Nanjing Normal University (Natural Science Edition)*, 42(02), 37-43.

ZENG Rong, ZHANG Cunbao, CAO Yu, et al. (2022). Evaluation method of traffic operation status of urban road intersection with efficiency and safety synergy-oriented. *Journal of Wuhan University of Technology (Transportation Science & Engineering)*, 46(06), 974-979.

Zhang Le. (2017). Research on urban signal intersection evaluation system. Chongqing Jiaotong University.

ZHAO Xing, LIN Hao, LIU Yi, et al. (2020). Comprehensive evaluation of signal control strategy of urban road intersection. *Journal of Chongqing Jiaotong University (Natural Science)*, 39(05), 7-13+31.

ZHENG X Y, DOU R, DING H, et al. (2011). Comprehensive Evaluation of Signalized Intersections Level-of Service Based on Information Entropy Theory. 11th International Conference of Chinese Transportation Professionals (ICCTP). New York: Curran Associates, Inc.