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“Controversies” in Special Relativity? Not So Fast!

Adrian Sfarti[1]

[1] CS Department, UC Berkeley, Soda Hall, CA 94720, USA
Correspondence: Adrian Sfarti, CS Department, UC Berkeley, Soda Hall, CA 94720, USA.

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Abstract

Even after 119 years from the discovery of special relativity now and then papers claiming to have discovered “controversies”: and “contradictions” in the theory surface occasionally. Dispelling such incorrect claims presents the opportunity to set things straight and to learn valuable lessons. In the present paper will present the correct, rigorous take on two such subjects: the shape of the spherical electromagnetic radiation wavefront (photon sphere) as viewed from a moving frame and the twin “paradox”.

Keywords: relativistic motion, the shape of the spherical electromagnetic radiation wavefront as viewed from a moving frame, the twin “paradox”

1. The Photon Sphere “Controversy”

In his paper (A. Vankov, n.d. a), A. Vankov discusses the case of a photon sphere as viewed from a frame F' in uniform motion with speed v with respect to the frame F in which the light source is resident. The author cites multiple sources, (R. Baierlein, 2006; Asher Peres, 1993; L. Sartori, 1984; A. Einstein, 1905; W. Pauli, 1981; J. Reitz, F. Milford & R. Cristy., 1993; J. Jackson, 1998; P. Tipler & R. Llewwellyn, 1999; W. Panofsky & M. Phillips, 1955; W. Rindler, 2001) amongst which A. Einstein (1905) is the first to make the claim that in both frames the wavefront is a sphere. Vankov sets to disprove the mainstream view by showing that, while in the “rest” frame the wavefront is a sphere of equation:

$$\begin{aligned} x &= (ct)\cos\theta\cos\phi \\ y &= (ct)\sin\theta\cos\phi \\ z &= (ct)\sin\phi \end{aligned} \tag{1}$$

in the “moving” frame, the wavefront is an ellipsoid contracted in the y direction (elongated in the x direction). Vankov derived this conclusion by applying the Lorentz transform to the time variable (correct) and the relativistic aberration formula to the angles θ, ϕ and this is where things went awry. The correct approach is to realize that θ, ϕ are parameters, not angles made by any light rays and, as such, they are not subject to aberration. In other words, the equation of the photon sphere in the rest frame can be written in a clearer way as:

$$
\begin{aligned}
x &= (ct)\cos q \cos f \\
y &= (ct)\sin q \cos f \\
z &= (ct)\sin f \\
x &= \gamma(x'-vt') \\
y &= y' \\
z &= z' \\
t &= \gamma(t'-\frac{vx'}{c^2})
\end{aligned} \tag{2}
$$

with *q, f* parameters, not angles that would be subject to aberration transformation. This results into the correct wavefront equation in the "moving" frame to be:

$$
\begin{aligned}
\gamma(x'-vt') &= [c\gamma(t'-\frac{vx'}{c^2})]\cos q \cos f \\
y' &= [c\gamma(t'-\frac{vx'}{c^2})]\sin q \cos f \\
z' &= [c\gamma(t'-\frac{vx'}{c^2})]\sin f
\end{aligned} \tag{3}
$$

as a direct result of the Lorentz transformation of (2). From the first equation of (3) we obtain:

$$x' = ct' \frac{\beta + \cos q \cos f}{1+\beta \cos q \cos f} \tag{4}$$

From (4) we obtain immediately:

$$t = \frac{t'}{\gamma(1+\beta \cos q \cos f)} \tag{5}$$

From (3), (4) and (5) we obtain:

$$
\begin{aligned}
y' &= ct' \frac{\sin q \cos f}{\gamma(1+\beta \cos q \cos f)} \\
z' &= ct' \frac{\sin f}{\gamma(1+\beta \cos q \cos f)}
\end{aligned} \tag{6}
$$

Armed with (4) and (6) we can calculate:

$$
\begin{aligned}
& x'^2 + y'^2 + z'^2 = \\
&= (ct')^2 \frac{(\beta + \cos q \cos f)^2 + (1-\beta^2)(\sin^2 q \cos f^2 + \sin^2 f)}{(1+\beta \cos q \cos f)^2} = \\
&= (ct')^2
\end{aligned} \tag{7}
$$

Come to think of this, this should have been obvious since the wavefront equation is a consequence of the electromagnetic wave equation. Since the latter equation is covariant, it follows that the former needs to be covariant as well, hence a sphere in both frames. In conclusion, Einstein's 1905 famous conclusion (A. Einstein, 1905), stands, there is no "controversy". The wavefront equation is:

$$x^2 + y^2 + z^2 = (ct)^2 \tag{8}$$

in frame F, while, in frame F′ is:

$$x'^2 + y'^2 + z'^2 = (ct')^2 \tag{9}$$

as derived by Einstein via a straight application of the Lorentz transforms and substituting the expressions for *x, y, z, t* into (8). It is interesting to note that while there is no aberration of the angles (parameters) *q, f,* there is "aberration" of the coordinates:

$$
\begin{aligned}
x' &= ct' \frac{\beta + \cos q \cos f}{1+\beta \cos q \cos f} \\
y' &= ct' \frac{\sin q \cos f}{\gamma(1+\beta \cos q \cos f)} \\
z' &= ct' \frac{\sin f}{\gamma(1+\beta \cos q \cos f)}
\end{aligned} \tag{10}
$$

2. The Twins' "Paradox" "Controversy"

Vankov argues (A. Vankov., n.d. a) that, in the absence of acceleration (at start of the trip, at the turning point and at twins' reunion) the twins' total elapsed proper time should be the same. This comes at odds with more detailed derivations that show Vankov (A. Vankov., n.d. a,b) to be on the wrong side of the argument. See, for details (A. Sfarti, 2012). Even if we simplified the thought experiment to eliminate acceleration, the difference in terms of the total elapsed proper times comes from the "jump" in the line of simultaneity at the turning point of the travelling twin. See Figure 1.

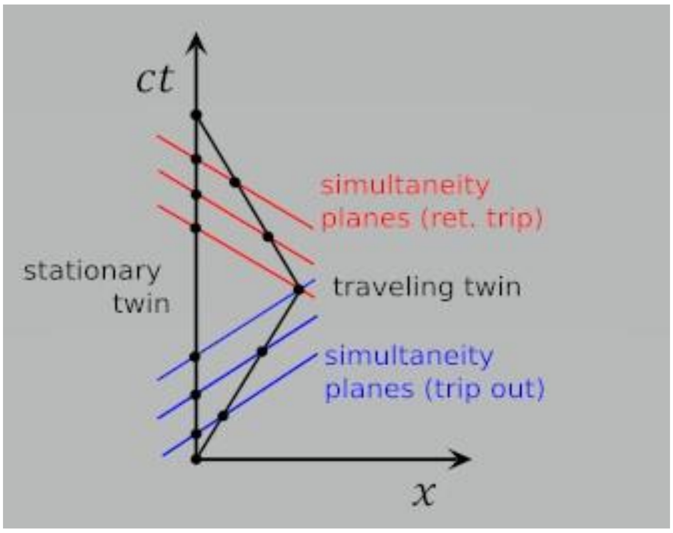

Figure 1. The twins' paths through spacetime

3. Conclusions

We have dispelled a couple of misconceptions about so-called "controversies": in the theory of relativity.

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