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# Approximation in a Nearring Using an Equivalence Relation with Thresholds

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**Abstract**

In this paper, we define an equivalence relation using level set of an i-v L-fuzzy ideal of nearing N. We use this equivalence relation to define upper and lower approximation of nonempty subset of the nearing N. We study properties of these approximations. We find relation between approximations in different cases.

**Keywords:** ideal, rough, approximation, fuzzy

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## 1. Introduction

Rosenfield (1971) initiated the study of fuzzy algebraic structures by introducing fuzzy subgroups. Pawlak (1982) introduced rough set theory. Ciric, Ignjatovic, Bogdanovic (2007) studied properties of equivalence classes of fuzzy equivalence relations over a complete residuated lattice and investigated fuzzy fuzzy partitions. Ignjatovic, Ciric, Bogdanovic (2009) defined fuzzy homomorphisms and used them as fuzzy congruences to relate elements of two possibly different algebras. Kedukodi, Kuncham and Bhavanari (2009) studied equiprime, 3-prime and c-prime fuzz ideals of nearrings and proved isomorphism theorems. Davvaz (2001) introduced interval valued L-fuzzy ideals of nearrings. Jagadeesha, Kuncham and Kedukodi (n.d.) introduced interval valued L-fuzzy ideals of nearring using interval valued t-norms and interval valued t-conorms. Kuncham, Jagadeesha and Kedukodi (n.d.) defined interval valued L-fuzzy cosets of a nearring and proved isomorphism theorems. Biswas and Nanda (1994) related algebraic structures and rough sets by substituting an algebraic system instead of the universe set. Davvaz (2006) initiated study of rough set theory based on fuzzy ideals. Kedukodi, Kuncham and Bhavanari (2010) proposed rough approximations which depend on a reference point. Zhang, Zhang and Wu (2009) proposed a general study of (I, T)-interval-valued fuzzy rough sets on two universes of discourse integrating the rough set theory with the interval-valued fuzzy set theory by constructive and axiomatic approaches. Shen and Wang (2011) initiated the construction of rough approximations of a vague set in fuzzy approximation space. Doina and Wang (2012) extended Pawlaks rough set theory to a topological model where the set approximations are defined using the topological notion $\alpha\beta$-open sets.

In this paper, we define a congruence relation using level set of an i-v L-fuzzy ideal of nearing N. We use this equivalence relation to define r approximation of nonempty subset of the nearing N. We study properties of these approximations. We find relation between approximations in different cases.

**2. Preliminaries**

In this paper $< L, \wedge, \vee >$ is a complete bounded lattice with greatest element M and least element m.

$\leq_L$ be the partial order in L. N, $N_1$ and $N_2$ will represent right nearrings. We refer to Gratzer (2011), Byth (2005), Birkoff (1995) for lattice, Klement, Mesiar and Pap (2000) for t-norms, and Pilz (1983), Ferrero and Ferrero (2002), Bhavanari and Kuncham (2013) for nearrings, Anderson and Fuller (1992) for rings, Pawlak (1982) for rough sets and Ciucci (2008) for rough approximation algebra, Klement, Mesiar and Pap (2000) for t-norm and t-conorms on a lattice, Davvaz (2001) for interval valued L-fuzzy sets. (Gu, Li, Chen & Lu, 1995) Let L be a lattice.

A t-norm is a function T : L × L → L such that ∀ x, y, z ∈ L the following axioms are satisfied: 1. Commutativity: T(x, y) = T(y, x), 2. Associativity: T(x, T(y, z)) = T(T(x, y), z), 3. Monotonicity: If $y \leq_L z$ then $T(x, y) \leq_L T(x, z)$, 4. Boundary condition: T(x, M) = x. Let T1 and T2 be two t-norms on L. If $T1(x, y) \leq_L T2(x, y)$ ∀ x, y ∈ L then we say that T1 is weaker than T2. We write $T1 \leq_L T2$ if T1 is weaker than T2. A t-norm T on L is called an idempotent t-norm if T (x, x) = x ∀ x ∈ L.

(Klement, Mesiar & Pap, 2000) Let L be a lattice. A t-conorm is a function C : L × L → L, such that ∀ x, y, z ∈ L the following axioms are satisfied: 1. Commutativity: C(x, y) = C(y, x), 2. Associativity: C(x, C(y, z)) = C(C(x, y), z), 3. Monotonicity: If $y \leq_L z$ then $C(x, y) \leq_L C(x, z)$, 4. Boundary condition: C(x, m) = x. Let C1 and C2 be two t-conorms on L. If $C1(x, y) \leq_L C2(x, y)$ ∀ x, y ∈ L then we say that C1 is weaker than C2. We write $C1 \leq_L C2$ if C1 is weaker than C2.

(Jagadeesha, Kuncham & Kedukodi, n.d.) Let (N,+,) be a nearring. Let $T_I$ be an i-v t-norm and $C_I$ be an i-v t-conorm on D(L). Let $\hat{\alpha}$, $\hat{\beta} \in$ D(L) with $\hat{\alpha} < \hat{\beta}$. An i-v L-fuzzy subset $\hat{\mu}$ on N is called an i-v L-fuzzy ideal with thresholds $\hat{\alpha}$, $\hat{\beta}$ if ∀ x, y, i ∈ N.

(1) $C_I(\hat{\alpha}, \hat{\mu}(x + y)) \geq T_I(\hat{\beta}, T_I(C_I(\hat{\alpha}, \hat{\mu}(x)), C_I(\hat{\alpha}, \hat{\mu}(y))))$,

(2) $CI(\hat{\alpha}, \hat{\mu}(-x)) \geq T_I(\hat{\beta}, CI(\hat{\alpha}, \hat{\mu}(x)))$,

(3) $C_I(\hat{\alpha}, \hat{\mu}(y + x - y)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(x)))$,

(4) $C_I(\hat{\alpha}, \hat{\mu}(xy)) \geq T_I(\hat{\beta}, CI(\hat{\alpha}, \hat{\mu}(x)))$,

(5) $C_I(\hat{\alpha}, \hat{\mu}(x(y + i) - xy)) \geq T_I(\hat{\beta}, CI(\hat{\alpha}, \hat{\mu}(i)))$

Definition. (Kazanci & Davvaz, 2008) Let θ be an equivalence relation on N, then equivalence class of x ∈ N is the set {y ∈ N | (x, y) ∈ θ} which is denoted by $[x]_\theta$.

Definition. (Kazanci & Davvaz, 2008) Let θ be an equivalence relation on N, then θ is called a full congruence relation if (a, b) ∈ θ and (c, d) ∈ θ implies (−a,−b) ∈ θ, (a + c, b + d) ∈ θ, and (ac, bd) ∈ θ for all a, b, c, d ∈ N. A full congruence relation is said to be complete if {xy | x ∈ [a]θ, y ∈ [b]θ} = [ab]θ for all a, b ∈ N.

(Kazanci & Davvaz, 2008) Let θ be a full congruence relation on N. If a, b ∈ N then, (i) $[a]_\theta + [b]_\theta = [a + b]_\theta$. (ii) $[-a]_\theta = -[a]_\theta$. (iii) $\{xy \mid x \in [a]_\theta, y \in [b]_\theta\} \subseteq [ab]_\theta$.

**3. Approximation in Nearing Using an Equivalence Relation with Thresholds**

**Definition**: Let $\hat{\mu}$ be an i-v L-fuzzy ideal of N. with associated i-v t-conorm $C_I$ and associated i-v t-norm $T_I$. Let $\hat{\alpha}$, $\hat{\beta} \in$ D(L) with $\hat{\alpha} < \hat{\beta}$. For $\hat{k} \in [\hat{\alpha}, \hat{\beta})$ define, $U(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta})$ $=\{(x,y) \in N \times N \mid T_I(C_I(\hat{\alpha}, \hat{\mu}(x-y))) \geq \hat{k}\}$. Then $U(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta})$ is called $\hat{k}$ level relation of $\hat{\mu}$ with thresholds $\hat{\alpha}$ and $\hat{\beta}$.

**Proposition**: Let $\hat{\mu}$ be an i-v L-fuzzy ideal of N. with associated i-v t-conorm $C_I$ and associated i-v t-norm $T_I$. Let $\hat{\alpha}$, $\hat{\beta} \in$ D(L) with $\hat{\alpha} < \hat{\beta}$. For $\hat{k} \in [\hat{\alpha}, \hat{\beta})$ define, $U(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta})$ $=\{(x,y) \in N \times N \mid T_I(C_I(\hat{\alpha}, \hat{\mu}(x-y))) \geq \hat{k}\}$. Then $U(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta})$ is an equivalence relation and full congruence relation on N.

**Notation**: Let $\hat{\mu}$ be an i-v L-fuzzy ideal of N. with associated i-v t-conorm $C_I$ and associated i-v t-norm $T_I$. Let $\hat{\alpha}, \hat{\beta} \in$ D(L) with $\hat{\alpha} < \hat{\beta}$. For $\hat{k} \in [\hat{\alpha}, \hat{\beta})$ define, U($\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}$) = {(x,y) $\in N \times N|$ $T_I$ $(C_I(\hat{\alpha}, \hat{\mu}(x-y))) \geq \hat{k}$}. Then U($\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}$) is an equivalence relation and full congruence relation on N. The equivalence class containing x is denoted by $[x]_{(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta})}$.

**Definition:** Let $\hat{\mu}$ be an i-v L-fuzzy ideal of N. with associated i-v t-conorm $C_I$ and associated i-v t-norm $T_I$. Let $\hat{\alpha}, \hat{\beta} \in$ D(L) with $\hat{\alpha} < \hat{\beta}$. For $\hat{k} \in [\hat{\alpha}, \hat{\beta})$ define, U($\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}$) ={(x,y) $\in N \times N|$ $T_I$ $(C_I(\hat{\alpha}, \hat{\mu}(x-y))) \geq \hat{k}$}. Then U($\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}$) is an equivalence relation and full congruence relation on N. Let X be a nonempty subset of N. Then the sets $U(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) = \{x \in N | [x]_{(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta})} \subseteq X\}$ and $\overline{U}(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) = \{x \in N | [x]_{(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta})} \cap X \neq \phi\}$ are respectively called upper and lower approximations of X with respect to U($\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}$). Then BND(X)= $\overline{U}(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) - U(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X)$ is called boundary region of X. If BND(X) not an empty set, then X is called a rough set otherwise called crisp set. If $U(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X)$ and $\overline{U}(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X)$ are ideals of N then (($U(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X)$, $\overline{U}(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X)$) is called rough ideal of N. If $\overline{U}(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X)$ is an ideal of N it is called upper rough ideal of N.

**Proposition**: If (N, U($\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}$)) is an approximation space and X is a subset of N then $U(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \subseteq \overline{U}(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X)$.

**Proposition**: Let $\hat{\mu}$ be an i-v L-fuzzy ideal of N with associated i-v t-conorm $C_I$ and associated i-v t-norm $T_I$. Let $\hat{\alpha}, \hat{\beta} \in$ D(L) with $\hat{\alpha} < \hat{\beta}$. Let X is an ideal of N. Then $\overline{U}(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X)$ is an upper rough ideal of N.

**Proposition**: Let $\hat{\mu}$ be an i-v L-fuzzy ideal of N with associated i-v t-conorm $C_I$ and associated i-v t-norm $T_I$. Let $\hat{\alpha}, \hat{\beta} \in$ D(L) with $\hat{\alpha} < \hat{\beta}$. Let X is a nonempty subset of N and $U(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \neq \phi$. Then $[0]_{(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta})} \subseteq X$.

**Proposition**: Let $\hat{\mu}$ be an i-v L-fuzzy ideal of N with associated i-v t-conorm $C_I$ and associated i-v t-norm $T_I$. Let $\hat{\alpha}, \hat{\beta} \in$ D(L) with $\hat{\alpha} < \hat{\beta}$. Let X is an ideal of N and $1 \in N$ and N has right inverse then $\overline{U}(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X)$ is an ideal of N.

**Proposition**: Let $\hat{\mu}$ be an i-v L-fuzzy ideal of N with associated i-v t-conorm $C_I$ and associated i-v t-norm $T_I$. Let $\hat{\alpha}, \hat{\beta} \in$ D(L) with $\hat{\alpha} < \hat{\beta}$. Let X is an ideal of N and $U(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \neq \phi$. Then $U(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X)$ =X.

**Proposition**: Let $\hat{\mu}$ be an i-v L-fuzzy ideal of N with associated i-v t-conorm $C_I$ and associated i-v t-norm $T_I$. Let $\hat{\alpha}, \hat{\beta} \in$ D(L) with $\hat{\alpha} < \hat{\beta}$. Let X is an ideal of N and $1 \in N$ and N has right inverse then ($U(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X)$, $\overline{U}(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X)$) is rough ideal of N.

**Proposition**: Let $\hat{\mu}$ be an i-v L-fuzzy ideal of N with associated i-v t-conorm $C_I$ and associated i-v t-norm $T_I$. Let $\hat{\alpha}, \hat{\beta} \in$ D(L) with $\hat{\alpha} < \hat{\beta}$. Let X is a nonempty subset of N. Let $\hat{\mu}$ and $\hat{\lambda}$ be i-v L-fuzzy ideals of N. Then

(1) $U(\hat{\mu} \cap \hat{\lambda}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \subseteq U(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \cap U(\hat{\lambda}, \hat{k}, \hat{\alpha}, \hat{\beta}, X)$.

(2) $\overline{\overline{U}}(\hat{\mu} \cap \hat{\lambda}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \supseteq \overline{\overline{U}}(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \cap \overline{\overline{U}}(\hat{\lambda}, \hat{k}, \hat{\alpha}, \hat{\beta}, X)$.

**Proposition**: Let $\hat{\mu}$ be an i-v L-fuzzy ideal of N with associated i-v t-conorm $C_I$ and associated i-v t-norm $T_I$. Let $\hat{\alpha}, \hat{\beta} \in$ D(L) with $\hat{\alpha} < \hat{\beta}$. Let $T_I$ be an i-v t-norm on D(L) and X is a nonempty subset of N. Let

$\hat{\mu}$ and $\hat{\lambda}$ be i-v L-fuzzy ideals of N. Then

(1) $\underline{U}(\hat{\mu} \cap_{T_I} \hat{\lambda}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \subseteq \underline{U}(\hat{\mu} \cap \hat{\lambda}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \subseteq \underline{U}(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \cap \underline{U}(\hat{\lambda}, \hat{k}, \hat{\alpha}, \hat{\beta}, X)$.
(2) $\overline{\overline{U}}(\hat{\mu} \cap_{T_J} \hat{\lambda}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \supseteq \overline{\overline{U}}(\hat{\mu} \cap \hat{\lambda}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \supseteq \overline{\overline{U}}(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \cap \overline{\overline{U}}(\hat{\lambda}, \hat{k}, \hat{\alpha}, \hat{\beta}, X)$.

**Proposition**: Let $\hat{\mu}$ be an i-v L-fuzzy ideal of N with associated i-v t-conorm $C_I$ and associated i-v t-norm $T_I$. Let $\hat{\alpha}, \hat{\beta} \in$ D(L) with $\hat{\alpha} < \hat{\beta}$. Let $\hat{\mu}$ and $\hat{\lambda}$ be i-v L-fuzzy ideals of N such that $\hat{\mu} \subseteq \hat{\lambda}$ and X is a nonempty subset of N. Then

(1) $\underline{U}(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \subseteq \underline{U}(\hat{\lambda}, \hat{k}, \hat{\alpha}, \hat{\beta}, X)$
(2) $\overline{\overline{U}}(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \supseteq \overline{\overline{U}}(\hat{\lambda}, \hat{k}, \hat{\alpha}, \hat{\beta}, X)$.

**Proposition**: Let $\hat{\mu}$ be an i-v L-fuzzy ideal of N with associated i-v t-conorm $C_I$ and associated i-v t-norm $T_I$. Let $\hat{\alpha}, \hat{\beta} \in$ D(L) with $\hat{\alpha} < \hat{\beta}$. Let X is a nonempty subset of N. Let $\hat{\mu}$ and $\hat{\lambda}$ be i-v L-fuzzy ideals of N. Then

(1) $\underline{U}(\hat{\mu} \cup \hat{\lambda}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \subseteq \underline{U}(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \cup \underline{U}(\hat{\lambda}, \hat{k}, \hat{\alpha}, \hat{\beta}, X)$.
(2) $\overline{\overline{U}}(\hat{\mu} \cup \hat{\lambda}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \supseteq \overline{\overline{U}}(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \cup \overline{\overline{U}}(\hat{\lambda}, \hat{k}, \hat{\alpha}, \hat{\beta}, X)$.

**Proposition**: Let $\hat{\mu}$ be an i-v L-fuzzy ideal of N with associated i-v t-conorm $C_I$ and associated i-v t-norm $T_I$. Let $\hat{\alpha}, \hat{\beta} \in$ D(L) with $\hat{\alpha} < \hat{\beta}$. Let $C_J$ be an i-v t-conorm on D(L) and X is a nonempty subset of N. Let $\hat{\mu}$ and $\hat{\lambda}$ be i-v L-fuzzy ideals of N. Then

(1) $\underline{U}(\hat{\mu} \cup_{C_J} \hat{\lambda}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \subseteq \underline{U}(\hat{\mu} \cup \hat{\lambda}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \subseteq \underline{U}(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \cup \underline{U}(\hat{\lambda}, \hat{k}, \hat{\alpha}, \hat{\beta}, X)$.
(2) $\overline{\overline{U}}(\hat{\mu} \cup_{C_J} \hat{\lambda}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \supseteq \overline{\overline{U}}(\hat{\mu} \cap \hat{\lambda}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \supseteq \overline{\overline{U}}(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \cup \overline{\overline{U}}(\hat{\lambda}, \hat{k}, \hat{\alpha}, \hat{\beta}, X)$.

**Proposition**: Let $\hat{\mu}$ be an i-v L-fuzzy ideal of N with associated i-v t-conorm $C_I$ and associated i-v t-norm $T_I$. Let $\hat{\alpha}, \hat{\beta} \in$ D(L) with $\hat{\alpha} < \hat{\beta}$. Let $\hat{v}$ and $\hat{\delta} \in$ D(L) such that $\hat{v} < \hat{\delta}$ and $\hat{\alpha} < \hat{v}$ and $\hat{\beta} < \hat{\delta}$. Then

(i) $\underline{U}(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \subseteq \underline{U}(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\delta}, X) \subseteq \underline{U}(\hat{\mu}, \hat{k}, \hat{v}, \hat{\delta}, X)$.
(ii) $\overline{\overline{U}}(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \supseteq \overline{\overline{U}}(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\delta}, X) \supseteq \overline{\overline{U}}(\hat{\mu}, \hat{k}, \hat{v}, \hat{\delta}, X)$.

**Definition:** Let $\hat{\mu}$ be an i-v L-fuzzy ideal of N. with associated i-v t-conorm $C_I$ and associated i-v t-norm $T_I$. Let $\hat{\alpha} = [m, m]$, $\hat{\beta} = [M, M] \in$ D(L) with $\hat{\alpha} < \hat{\beta}$. Let $\hat{k} \in [\hat{\alpha}, \hat{\beta})$. Let X be a nonempty subset of N. Then the sets $\underline{U}(\hat{\mu}, \hat{k}, X) = \{x \in N | [x]_{(\hat{\mu}, \hat{k})} \subseteq X\}$ and $\overline{U}(\hat{\mu}, \hat{k}, X) = \{x \in N | [x]_{(\hat{\mu}, \hat{k})} \cap X \neq \phi\}$ are respectively called upper and lower approximations of X with respect to U($\hat{\mu}$, $\hat{k}$). Then BND(X)= $\overline{U}(\hat{\mu}, \hat{k}, X) - \underline{U}(\hat{\mu}, \hat{k}, X)$ is called boundary region of X. If BND(X) not an empty set, then X is called a rough set otherwise called crisp set.

**Proposition**: Let $\hat{\mu}$ be an i-v L-fuzzy ideal of N with associated i-v t-conorm $C_I$ and associated i-v t-norm $T_I$. Let $\hat{\alpha}, \hat{\beta} \in$ D(L) with $\hat{\alpha} < \hat{\beta}$. Let $\hat{\mu}$ and $\hat{\lambda}$ be i-v L-fuzzy ideals of N such that $\hat{\mu} \subseteq \hat{\lambda}$ and X is a nonempty subset of N. Then

(i) $\underline{U}(\hat{\mu}, \hat{k}, X) \subseteq \underline{U}(\hat{\mu}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \subseteq \underline{U}(\hat{\mu}, \hat{k}, \hat{\alpha}, [M,M], X)$.
(ii) $\overline{\overline{U}}(\hat{\mu}, \hat{k} X) \supseteq \overline{\overline{U}}(\hat{\lambda}, \hat{k}, \hat{\alpha}, \hat{\beta}, X) \supseteq . \overline{\overline{U}}(\hat{\mu}, \hat{k}, \hat{\alpha}, [M, M], X)$

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