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Exploring the Application of High-Road Transfer in the Teaching of "Double Angle Bisector Modeling" Problem Solving

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Abstract

In the current teaching practice of junior middle school mathematics, teachers tend to directly teach the conclusion of geometric models, focusing on the training of low-road transfer, which leads to students' difficulty applying the model flexibly to solve problems. In order to change this situation, the teaching of geometric model problem solving should be optimized based on the law of high-road transfer. Specific measures include: digging deep into the principles behind the model to help students identify the general principles and logical structure; establishing a model thinking framework by sorting out the internal logic of the mathematical model; and designing variations of practice questions to consolidate and deepen students' understanding and to achieve the ability of high-road transfer. In this paper, for the problem-solving teaching of "double angle bisector modeling", we give a high-road transfer path, refine the core ideas of the model and establish the solution ideas of related problems, so as to provide reference for the optimization of junior high school mathematics problem-solving teaching.

Keywords: high-road transfer, teaching problem solving, double angle bisector modeling

1. Foreword

Many geometric problems in junior secondary mathematics require the use of geometric models, the selection of appropriate models based on the geometric features they present, and the application of uniform problem-solving procedures or the use of model conclusions to assist in solving the problems. Among them, the use of the "double angle bisector modeling" is particularly important. However, the current teaching mode of "direct memorization and repetition" can help students cope with some of the standard problems in the short term, but when faced with complex and changing geometric problems, students often find it difficult to comfortably apply the "direct memorization and repetition" model to solve them. However, when faced with complex and changing geometric problems, students often find it difficult to use the "double angle bisector modeling" freely, and are at a loss as to the selection and application of the model.

The high-road transfer theory in educational psychology provides a new perspective for solving this problem, which focuses on extracting universally applicable principles and methods from specific situations, so as to realize effective transfer of knowledge in new situations. Integrating the theory of high-road transfer into the problem-solving teaching of "double angle bisector modeling" goes beyond the mechanical memorization of simple conclusions, and digs deep into the intrinsic principles and logical structure of the model, guiding students to build a systematic problem-solving strategy and thinking mode on the basis of understanding the essence of the model.

The optimization of the teaching of the "double angle bisector modeling" aims to enhance the teaching effect and learning quality through the guidance of the high-road transfer theory. Firstly, the principles behind the model are explored in depth to help students identify the general principles and logical structure; secondly, the internal logic of the mathematical model is sorted out to establish a framework for the model; finally, variations of practice questions are designed to consolidate and deepen students' understanding, and to achieve efficient transfer and flexible application of knowledge.

2. Exploring the Principles of Modelling and Laying the Foundations for Migration

In junior secondary mathematics teaching, teachers should help students tap into mathematical principles based on their original cognitive starting points through the strategy of organizing and paving, so as to lay the foundation stone for their effective transfer in the learning process. (Liu Xiuli & Zhang Zhongqiang, 2017) There are a total of three basic forms of the "double angle bisector modeling", which are divided according to the type of angle bisected to derive the quantitative relationship between the two angles. (Tao M., 2018) The basic model is the one that is most often used in problem solving. It is straightforward to find the corresponding model based on the graph, and then quickly derive the quantitative relationship between the angles. By analyzing the connection between the derivation processes, it is found that all three models can be derived by equations and elimination.

2.1 The Angle Between the Bisectors of Two Interior Angles of a Triangle

The model represents the quantitative relationship between the angle between the angle bisectors of the two interior angles of a triangle and the other angle of that triangle, and the conclusion of the model should be familiarised with, i.e. $\angle P = 90° + \frac{1}{2}\angle A$.

[Given] As shown in Figure 1, in ΔABC in which BP bisect $\angle ABC$ and CP bisect.

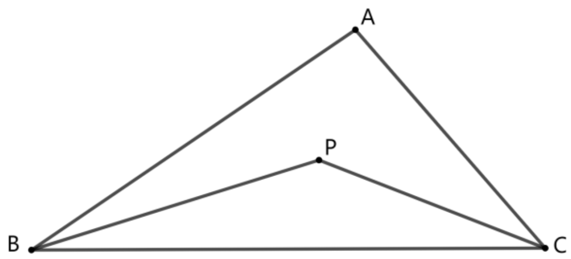

Figure 1.

[Conclusion] $\angle P = 90° + \frac{1}{2}\angle A$.

[Derivation] As in Figure 1, let $\angle PBC = x$, $\angle PCB = y$, $\because$ BP bisect $\angle ABC$ and CP bisect $\angle ACB$, $\therefore \angle ABC = 2x$, $\angle ACB = 2y$, $\because$ In ΔPBC, $\angle P + \angle PBC + \angle PCB = 180°$, in ΔABC. $\angle A + \angle ABC + \angle ACB = 180°$,

$$\therefore \begin{cases} \angle P + x + y = 180° & ① \\ \angle A + 2x + 2y = 180° & ② \end{cases}.$$

Method 1 (addition and subtraction of elimination elements): ② − 2①, $\therefore \angle P = 90° + \frac{1}{2}\angle A$, the proof is complete.

Method 2 (substitution and elimination): from ① is obtained, $x + y = 180° - \angle P$, and bring in ②, $\angle P = 90° + \frac{1}{2}\angle A$, and the proof is complete.

2.2 The Angle Between the Bisectors of Two Exterior Angles of a Triangle

The model represents the quantitative relationship between the angle between the angle bisectors of the two exterior angles of a triangle and the other angle of that triangle, and the conclusion of the model should be familiarized with, i.e. $\angle P = 90° - \frac{1}{2}\angle A$.

[Given] As shown in Figure 2, in ΔABC in which BP bisect $\angle DBC$ and CP bisect $\angle ECB$.

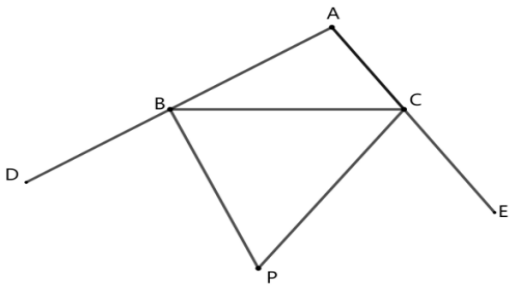

Figure 2.

[Conclusion] $\angle P = 90° - \frac{1}{2}\angle A$.

[Derivation] As in Figure 2, let $\angle PBC = x$, $\angle PCB = y$, and $\because$ BP bisect $\angle DBC$ and CP bisect $\angle ECB$, $\therefore \angle ABC = 180° - 2x$, $\angle ACB = 180° - 2y$.

In ΔPBC, $\angle P + \angle PBC + \angle PCB = 180°$, In ΔABC, $\angle A + \angle ABC + \angle ACB = 180°$,

$$\therefore \begin{cases} \angle P + x + y = 180° & ① \\ \angle A + (180° - 2x) + (180° - 2y) = 180° & ② \end{cases}$$

Method 1 (addition and subtraction of elimination elements): ② + 2①, $\angle P = 90° - \frac{1}{2}\angle A$, the proof is complete.

Method 2 (substitution and elimination): from ① is obtained, $x + y = 180° - \angle P$, and bring in ②, $\angle A + 360° - 2(180° - \angle P) = 180°$, and collate to get $\angle P = 90° - \frac{1}{2}\angle A$, and the proof is complete.

2.3 The Angle Between an Interior Angle and the Bisector of an Exterior Angle of a Triangle

The model represents the quantitative relationship between the angle between the angle bisector of an interior angle and the angle bisector of an exterior angle of a triangle and the other angle of that triangle, and one should familiarize oneself with the conclusion of the model, i.e. $\angle P = \frac{1}{2}\angle A$.

[Given] As shown in Figure 3, in ΔABC in which BP bisect $\angle ABC$ and CP bisect $\angle ACD$.

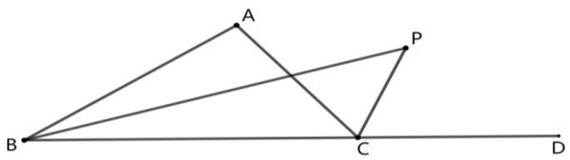

Figure 3.

[Conclusion] $\angle P = \frac{1}{2}\angle A$.

[Derivation] As in Figure 3, let $\angle PBC = x$, $\angle PCD = y$, $\because$ BP bisect $\angle ABC$ and CP bisect $\angle ACD$, in ΔPBC, $\angle P + \angle PBC + \angle PCB = 180°$, In ΔABC, $\angle A + \angle ABC + \angle ACB = 180°$.

$$\therefore \begin{cases} \angle P + x + (180° - y) = 180° & ① \\ \angle A + 2x + (180° - 2y) = 180° & ② \end{cases}$$

Method 1 (addition and subtraction of elimination elements): ② − 2①, the proof is complete.

Method 2 (substitution and elimination): from ① is obtained, and bring in ②, $\angle A + 180° - 2\angle P = 180°$, and collate to get $\angle P = \frac{1}{2}\angle A$. and the proof is complete.

Analysis of the principles of geometric modelling is essential. In the teaching of "double angle bisector model", the teacher should introduce students to the three basic forms of the model: "double angle bisector modeling", "double exterior angle bisector angle type" and "one interior and one exterior angle bisector type". For each form, the teacher should analyze the derivation process in detail so that students can clearly see how the model evolves from simple to complex and from single to multiple. In addition, the teacher can also guide students to compare "double angle bisector modeling" with other models to discover the commonalities and differences between them, so as to better grasp the application of the model.

3. Conceptualizing the Model Pulse and Mastering the Migration Vector

The body of knowledge in junior secondary mathematics is closely related to each other, and the similarity between model principles is something that teachers and students need to grasp. The process of establishing a migration relationship between model ideas and new problems, and solving problems by touch, needs to be discovered by both teachers and students, i.e. teachers should guide students to know what type of topics to use the model (premise) when encountering them, as well as the specific process of applying the model to solve problems.

The derivation process of the three models follows the same framework of ideas, i.e. equations → permutations x and y → elimination, the essence of which is to is divided into n equal parts, the two smallest angles formed after equipartition establishes a link between the two equations, how many equal parts of the angle, equal parts of the interior or exterior angles, will not affect the idea of this question, therefore, the model will be similar preconditions with the model to establish a unified approach to the problem steps:

[Prerequisite] two angles, n-equilibrium line.

[Steps] There are three steps, as shown in table 1:

Table 1. Double angle bisector modeling idea doing steps and ideas

workflows	**frameworks**	**exegesis**	**mathematical thought**
step 1	equations	① Two equations based on the sum of the interior angles of two triangles is 180° respectively. ② Some questions may require equations using the sum of the interior angles of an n-sided shape.	mathematical thinking
step 2	change x sum y	① Let the degrees of the two smallest angles obtained after n equal divisions be x and y respectively. ② Express all angles in the system of equations other than ∠A and ∠P in terms of x and y.	holistic thinking; eliminate one variable from equations
step 3	eliminate one variable from equations	① By replacing the system of equations x and y operations as a whole. ② Use additive and subtractive elimination or carry-in elimination to combine the values containing x and y and the whole operation of eliminating.	

The table summarizes the steps of the double angles and equidistant lines questions according to the derivation of the three basic models. Equidistant line problem type of doing the steps, you can follow the above three steps to establish the framework for the solution of the relevant types of problems. The similarity between models is crucial to solving problems. The knowledge systems of junior middle school mathematics are closely linked, and in teaching, teachers should pay attention to cultivating students' analogy and transfer ability, and help students construct perfect solution ideas by guiding them to discover the potential relationship between similar models. The cultivation of this ability not only helps students to quickly find the right direction in the process of solving problems and establish the specific process of applying models to solve problems, but also improves their problem-solving speed and accuracy.

4. Apply Modelling to Solve Problems and Promote Flexible Transfer

This link can be used in the method of variant teaching. Variation teaching is one of the most important means to enable students to grasp concepts exactly, that is, to use different forms of visual materials or examples in teaching to clarify the essential properties of things, so as to form a scientific concept of a thing. (Li Jing, 2016) The core of variant teaching is to show different faces of the same concept through diversified examples, so as to encourage students to identify and understand the common principles behind these examples, and then to achieve a deeper and high-road transfer. In the following, we will introduce the process of applying the model thinking framework to solve problems through four variant topics.

4.1 Variant 1

As shown in Figure 4, in ΔABC, ∠A = 72°, the trisectors of ∠ABC and ∠ACB intersect at the point P, finds the degree of ∠P.

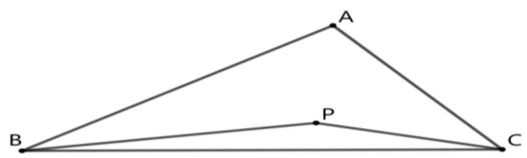

Figure 4.

[Ideas] This question fulfills the prerequisite that two angles are divided by n equals, and therefore can be based on the three-step method of Table 1, listed in the problem solution ideas framework, as shown in Table 2:

Table 2. Solution framework for Variation 1

workflows	frameworks	rig	exegesis
step 1	equations	$\begin{cases}\text{In } \Delta\text{PBC}，\angle\text{P}+\angle\text{PBC}+\angle\text{PCB}=180° & ① \\ \text{In } \Delta\text{ABC}，\angle\text{A}+\angle\text{ABC}+\angle\text{ACB}=180° & ②\end{cases}$	Utilize ΔPBC and ΔABC the sum of the interior angles of two triangles is 180° to create the system of equations.
step 2	change x sum y	Set up$\angle\text{PBC}=x$ set up $\angle\text{PCB}=y$ $\therefore\ \angle\text{ABC}=3x,\ \angle\text{ACB}=3y$ $\therefore$ There are systems of equations $\begin{cases}\angle\text{P}+x+y=180° & ① \\ \angle\text{A}+3(x+y)=180° & ②\end{cases}$	Let the degrees of the two smallest angles formed after trisection, ∠PBC and ∠PCB , be x and y respectively, and the degrees of ∠ABC and ∠ACB be expressed in terms of x and y as well.
step 3	eliminate one variable from equations	Method 1 (addition and subtraction of elimination elements): ② − 3① Gain. $\angle\text{P}=\frac{1}{3}(360°+\angle\text{A})=144°$	See $x+y$ as a whole.
		Method 2 (substitution and elimination): from① is obtained. and bring in② and collate to get$\angle\text{P}=\frac{1}{3}(360°+\angle\text{A})=144°$	

4.2 Variant 2

As in Figure 5, $\angle\text{PBC}=\frac{1}{3}\angle\text{DBC},\ \angle\text{PCB}=\frac{1}{3}\angle\text{ECB},$ explore the relationship between ∠A and ∠P.

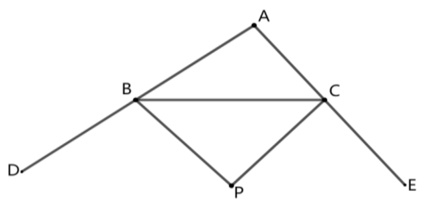

Figure 5.

[Ideas] This question fulfils the prerequisite that two angles are divided by n equals, and therefore can be based on the three-step method of Table 1, listed in Table 3, the problem solving ideas framework:

Table 3. Solution framework for Variation 2

workflows	frameworks	rig	exegesis
step 1	equations	$\begin{cases}\text{In } \Delta\text{PBC}，\angle\text{P}+\angle\text{PBC}+\angle\text{PCB}=180° & ① \\ \text{In } \Delta\text{ABC}，\angle\text{A}+\angle\text{ABC}+\angle\text{ACB}=180° & ②\end{cases}$	Utilize ΔPBC and ΔABC the sum of the interior angles of two triangles is 180° to create the system of equations.
step 2	change x	Set up$\angle\text{PBC}=x$, set up $\angle\text{PCB}=y$	Let the degrees of the two smallest angles formed

	sum y	$\therefore\ \angle ABC = 180° - 3x,\ \angle ACB = 180° - 3y$ $\therefore$ There are systems of equations $\begin{cases} \angle P + x + y = 180° & ① \\ \angle A + (180° - 3x) + (180° - 3y) = 180° & ② \end{cases}$	after trisection, ∠PBC and ∠PCB, be x and y respectively, and the degrees of ∠ABC and ∠ACB be expressed in terms of x and y as well.
step 3	eliminate one variable from equations	Method 1 (addition and subtraction of elimination elements): ②+3① gain, $\angle P = \frac{1}{3}(360° - \angle A)$. Method 2 (substitution and elimination): from① is obtained. and bring in② and collate to get$\angle P = \frac{1}{3}(360° - \angle A)$.	See $x + y$ as a whole.

4.3 Variant 3

As in Figure 6, in the plane right-angled coordinate system. The vertices A and B of ΔAOB are on the x − axis and y − axis respectively , and the bisector of ∠OAB intersects the inverse prolongation of the bisector of ∠ABy at point P, Find the degree of ∠P.

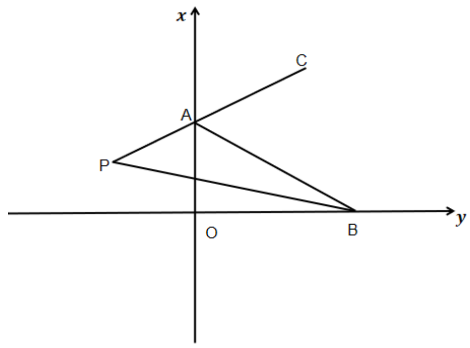

Figure 6.

[Ideas] This question fulfils the prerequisite that two angles are divided by n equals, and therefore can be based on the three-step method of Table 1, listed in the problem solution ideas framework, as shown in Table 4:

Table 4. Solution framework for Variation 3

workflows	**frameworks**	**rig**	**exegesis**
step 1	equations	$\begin{cases} \text{In } \Delta PAB, \angle P + \angle PAB + \angle PBA = 180° & ① \\ \text{In } \Delta ABO, \angle BOA + \angle OAB + \angle OBA = 180° & ② \end{cases}$	Utilise ΔPAB and ΔABO the sum of the interior angles of the two triangles is 180° to set up the system of equations.
step 2	change x sum y	Set up$\angle ABC = x$, set up$\angle PAB = y$ $\therefore\ \angle PBA = 180° - x,\ \angle OAB = 2y$ $\therefore$There are systems of equations $\begin{cases} \angle P + y + (180° - x) = 180° & ① \\ \angle A + 2y + (180° - 2x) = 180° & ② \end{cases}$	Let the degrees of the two smallest angles formed after bisection, ∠ABC and ∠PAB, be x and y respectively, and the degrees of ∠PBA, ∠OBA and ∠OAB be expressed in terms of x and y as well.

step 3	eliminate one variable from equations	Method 1 (addition and subtraction of elimination elements): ② − 2① gain, $\angle P = \frac{1}{2}\angle AOB$. Method 2 (substitution and elimination): from ① is obtained. and bring in ② and collate to get $\angle P = \frac{1}{2}\angle AOB$.	See $x - y$ as a whole.

4.4 Variant 4

As in Figure 7, in quadrilateral MNCB, BD bisects ∠MBC and intersects with the exterior angle of quadrilateral MNCB ∠NCE the angle bisector of quadrilateral MNCB intersects at point P, if ∠BMN = 130°, ∠CNM = 100°, find the degree of ∠D.

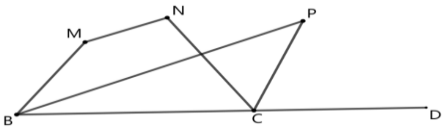

Figure 7.

[Ideas] This question fulfils the prerequisite that two angles are divided by n equals, and therefore can be based on the three-step method of Table 1, listed in the problem solution ideas framework, as shown in Table 5:

Table 5. Solution framework for Variation 4

workflows	frameworks	rig	exegesis
step 1	equations	$\begin{cases} \text{In } \Delta PBC, \angle P + \angle PBC + \angle PCB = 180° & ① \\ \text{In quadrilateral MNCB}, \angle M + \angle N + \angle NCB + \angle MBC = 360° & ② \end{cases}$	First create a system of equations using the sum of the interior angles of the quadrilateral MNCB as 360° and the interior angles of the triangle ABC as 180.
step 1	change x sum y	Set up$\angle PBC = x$, set up$\angle PCE = y$ $\therefore \angle MBC = 2x,\ \angle PCB = 180° - y$ ∴There are systems of equations $\begin{cases} \angle P + x + (180° - y) = 180° & ① \\ 130° + 100° + 2x + (180° - 2y) = 360° & ② \end{cases}$	let the degrees of the two smallest angles formed by bisecting ∠PBC and ∠PCE be x and y respectively, and the degrees of ∠MBC , ∠NCB and ∠PCB be expressed in terms of x and y as well.
step 1	eliminate one variable from equations	Method 1 (addition and subtraction of elimination elements): ② − 2① gain. $\angle P = \frac{1}{2} \times 50° = 25°$. Method 2 (substitution and elimination): from① is obtained. and bring in② and collate to get $\angle P = \frac{1}{2} \times 50° = 25°$.	See $x - y$ as a whole.

Variable training also plays an important role in the teaching process. By designing diversified practice questions and practical problems, teachers can help students consolidate what they have learnt, broaden their ideas for solving problems and cultivate the ability to think from multiple perspectives. In the variant training, teachers should pay attention to the hierarchy and difficulty gradient of the questions, so that students can gradually deepen their understanding and application of what they have learnt, and through the construction of clear ideas for solving the problems, so that students can better understand and apply what they have learnt and improve their ability to solve the problems. In the process of solving problems, teachers can guide students to use the elimination methods of addition, subtraction and substitution to simplify complex problems into simple ones. At the same time, teachers should also guide students to pay attention to the fact that the difference of certain angles can be regarded as a whole for calculation in order to simplify the calculation process.

5. Summaries

Based on the theory of high-road transfer, the above discusses in depth how teachers can help students optimize their learning of mathematics by teaching “double angle bisector modeling” in junior middle school mathematics. This model is an important model in junior high school geometry learning, which involves the bisector of angles, the properties of triangles and other knowledge point. In order to enable students to better master this model, teachers should start from the basic principles of the model and help students deeply understand its connotation and substance. In conclusion, the high-road transfer in learning migration plays a key role in the teaching of mathematical problem solving. In the teaching of junior high school mathematics, teachers should pay attention to cultivating students’ learning migration ability, through digging the principles of the model, constructing problem solving ideas, designing variant training, etc., to help students better master and apply what they have learnt, improve their problem solving ability and mathematical literacy, and lay a solid foundation for students’ future learning and development.

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