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Research on Mathematical Concept Teaching Strategies in Senior High School Based on the APOS Theory

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Abstract

Mathematical concepts are the basic elements of mathematics and the key for students to understanding and mastering problem-solving skills along with mathematical thoughts, so they are critical to learning and teaching mathematics. However, current mathematical concept teaching in senior high school is facing various challenges. For example, many teachers ignore the background against which these concepts were developed and fail to lead students to explore the essence of concepts, intentionally memorize mathematical concepts, or figure out their structure. The concept teaching based on the APOS (action, process, object, and schema) theory now prevails. This paper explores the strategies of teaching mathematical concepts in senior high school using the APOS theory, so as to provide a reference for teachers who also engage themselves in this field.

Keywords: concept teaching, APOS theory, constructivism, concept of function

1. Introduction

The *General High School Mathematics Curriculum Standards (2017)* emphasizes that the ultimate goal of mathematics teaching is to cultivate students' mathematical abilities, and they shall obtain rational cognition of mathematics concepts' nature via this activity (Ministry of Education of the People's Republic of China, 2017). All mathematical propositions, theorems, and formulas, alongside mathematics teaching, start with concepts, the study of which helps develop students' abstract thinking skills and other literacy. Therefore, how to better teach mathematical concepts in high school has become a hot topic in the mathematics education research field.

Current mathematical concept teaching in high school has four obvious problems. Firstly, teachers spend little time in classroom introduction and seldom or even never instruct students on the background of a concept. Secondly, teachers focus more on whether students have learned the concepts instead of allowing them to explore the essential characteristics of concepts. Thirdly, students spend more time doing exercises but fail to memorize the concepts intentionally. Finally, few teachers would summarize the knowledge scattered in the textbooks. All of these problems hinder students' mastery of the essential concept

features and the development of their learning abilities. Therefore, teachers urgently need to improve the quality of concept teaching.

To this end, many mathematics education experts and scholars have devoted themselves to finding appropriate concept teaching models or theories. One is the "5E" (engage, explore, explain, elaborate, and evaluate) instructional model. In literature (Xiang Xiannian, 2020), the author believes that it can help students learn concepts more efficiently and promote their literacy. This model has not been promoted in practical teaching yet due to some limitations. Another one is "Scaffolding Instruction". Literature (Zhang Junlian, 2015) holds the view that the concept teaching under the scaffolding model has improved the way students learn concepts, making them more interested and proactive in such an activity. However, some issues still need to be explored in depth, such as determining the heights that students can reach with the assistance of teachers and dealing with differences in students' learning abilities. The APOS theory is also notable. In literature (Yi Huaichu, 2020), the author believes that using it reasonably can effectively improve the teachers' teaching style. Guided by this theory, teachers take the students as the main body and lead them in joint inquiry and reflection. At this time, students can independently construct a mathematical knowledge system, enhance their understanding level, and thus better construct their mathematical ideas.

To sum up, APOS-based concept teaching can better cultivate students' independent inquiry ability and innovative spirit as well as facilitate concept teaching. Therefore, this paper focuses on the four stages of APOS theory and proposes concept teaching strategies, aiming to provide a reference for front-line teachers.

2. APOS Theory

The APOS theory was proposed by the American mathematics educator Dubinsky and his fellows at the end of the last century, which focuses on the cognition of individuals in constructing mathematical concepts and is related to mathematical concept learning and students' cognitive development (Jiang Chunlian, Hu Ling, 2020). This theory indicates that individual mental construction of mathematical concepts specifically goes through four stages: action, process, object, and schema. "Action" is the introduction of the concept. This stage emphasizes the educational value of "facts" by asking students to experience the development background of mathematical concepts and to perceive the connection between mathematics and real life. "Process" represents the definition. This stage requires students to describe and reflect on external activities and then summarize their common features, so as to build a thinking framework in minds and finally abstract the qualities of the concept. The "object" stage is concept analysis. This stage involves studying the concept formation as a whole object, assigning formal definitions and symbols to this object, establishing relationships between concept properties and attributes, and enabling students to deepen their understanding of concepts. "Schema", the final stage of mathematical concept learning, requires students to integrate what has been learned after repeating previous stages. They can use auxiliary tools such as mind maps to connect the newly learned knowledge and create a conceptual network in their minds.

3. Problems

Mathematical concepts reflect the essential properties of numbers and shapes among objective things, and at the same time are the soul and essence of the mathematics subject. Mathematical concept teaching should be student-oriented and guided by teachers. As mathematical concepts are highly abstract, so this paper mainly studies how teachers can promote students' understanding of these concepts. We first analyzed problems in practical concept teaching.

3.1 Teachers Ignore the Background of Mathematical Concepts

Many teachers can realize that how a concept emerges is important to students in the context of exam-oriented education, but they worry that exploring this process will take time out of the teaching schedule and would rather directly tell students what they will learn. However, senior high school freshmen are weak in abstraction, and they can hardly get interested in learning concepts if they don't know how a concept is formulated. Therefore, teachers should reserve enough time for introducing and helping students better understand the background of concepts.

3.2 Teachers Undervalue the Inquiry into the Essential Properties of Mathematical Concepts

Traditionally, some mathematics teachers

focused on the acquisition of a concept while failing to use real-life examples to help students research and reflect, and they never or seldom led students to explore the essential properties of concepts. As a result, students could not get an in-depth understanding of these concepts or could only memorize them mechanically. In addition, teachers tended to prioritize their own roles and textbooks, rendering student passive in building concepts and hindering the development of their independent learning capabilities. Therefore, in the "process" stage, teachers should be student-centered, pay attention to the process of discovering the essential properties of concepts, and promote students' transition from perceptual to rational understanding.

3.3 Students Fail to Intentionally Memorize Mathematical Concepts

Learning mathematical concepts is never a short process. After acquiring a mathematical concept, students still need to analyze and understand it in depth, and then refine its essence. In reality, however, teachers are used to leaving most of the time for students to do exercises as a supplement and claiming that the concept is a definite conclusion. As such, students don't have enough time to actively and deeply reflect on the newly learned mathematical concept, and their cognition of this concept turns out to be one-sided. Solving real-life problems with this concept will not be easy for them since they cannot analyze problems in an all-round way. Therefore, teachers should pay attention to deepening students' intentional memory of mathematical concepts, allowing them to actively explore, and developing their thinking skills.

3.4 Students Cannot Figure out the Structure of Mathematical Concepts

"Schema" requires students to figure out the connections between concepts, form a conceptual system, and build a comprehensive schema in the mind. However, some teachers themselves do not realize that the construction of a mathematical concept system facilitates concept understanding, so mathematical concepts are not taught alongside their structure. In this way, students would easily confuse concepts since they are unclear about the relationship between them. Thereupon, teachers should highlight such a structure, ask students to figure out their inner laws and connections and construct a net-like schema in the mind, cultivate students' ability to construct a conceptual system, and deepen their understanding of mathematical concepts' knowledge structure.

4. Strategies

The APOS theory put forward by Dubinsky can help solve these four problems. It takes the thinking process of students' active exploration as the starting point, which is followed by four stages (action, process, object, and schema), helping students to build a complete knowledge framework, cultivate thinking ability, and develop mathematical thinking methods. We thereby propose the following strategies corresponding to these four stages.

4.1 Create Situations Where Students Can Experience How a Mathematical Concept Forms

The "self-reference effect" suggests that learners can better remember what they have experienced firsthand. Before learning a concept, students have a certain knowledge base and rely little on the textbook, so they can be active in their study. Therefore, in the "action" stage, teachers need to collaborate with students, introduce the background of concepts based on students' existing cognition, set up real or virtual situations to stimulate students' senses, and guide them to perceive and comprehend the background against which a mathematical concept form by themselves.

Take teaching the "concept of function" in senior high school as an example. First of all, before introducing the concept, teachers might as well ask students to review the related knowledge they have learned in junior high school. This helps them build a bridge between new and previous knowledge and makes students wonder why they should redefine the concept of function in senior high school. As they may doubt what they learned before, they would be eager to find the secrets behind. Second, teachers can come up with a question: Is a function? Or "Is and the same function"? By thinking about these questions, students would realize that what they have learned in junior high school cannot solve these questions and they need to redefine the concept. Thus, students are involved in the background of learning a new concept, stimulating their desire to explore, and then laying a foundation for introducing the concept of function.

4.2 Design a Series of Questions that Enable

Students to Explore the Essential Attributes of Mathematical Concepts

Vygotsky, the famous Russian psychologist, believes that the gap between the students' current level and the possible development level is the zone of proximal development. Therefore, in the "process" stage, teachers are required to design a series of inspiring questions based on the textbook, let students explore and reflect, and stimulate students' interest in learning. Then they can internalize the mathematical concepts in their minds (Chen Shu, 2019). For example, when teaching the concept of function, teachers can choose four situations close to our daily life: a high-speed train trip, a worker's salary, a graph of the changing air quality index in Beijing on a certain day, and a table of the Engel coefficient of urban residents in a Chinese province. When discussing the first situation, teachers can raise these questions:

(1) Is the distance a function of time?

(2) Can you tell me what is the corresponding distance the train travels in one hour based on the existing conditions, and if you cannot, why?

(3) What is the variation range of time?

(4) How do you think the variation range of the independent variable should be described to reflect this situation?

(5) How can we use a set to describe the value range of an independent variable?

Through the above questions, the teacher guides students to focus on the variation range of the independent variable and gradually introduces the language of sets.

As for the other three situations, the teacher can also raise several exploratory questions and ask students to consider what common features are shared by the functions in these situations and whether they can abstract the essential features of the function's concept. Students can easily discover the essential properties of the concept via a probe into these questions. During this period, they are intellectually and emotionally involved, and they gradually become independent inquirers, successfully completing the transition from perceptual to rational understanding.

4.3 Devise Question Variants to Deepen Students' Understanding of Mathematical Concepts

To find the essential features of things from various angles, levels, and aspects, as well as to find their implied factors, we can first analyze the non-essential features of the same class of things, then change the perspectives and methods of observing things. Students can do so to understand mathematical concepts more deeply (Ling Jianqing, 2021). Therefore, in the "object" stage, teachers need to provide students with variants of questions in different contexts in order to identify and consolidate their understanding of the basic concept properties.

For example, students may find it difficult to understand the meaning of function symbols when studying the concept of function. At this time, teachers can design question variants to help students make a breakthrough. Specifically, some students cannot identify the difference and connection between and, and teachers can help them solve this puzzle by setting these two questions:

(1) Given the function, please work out the value of.

(2) When, what are the values of and?

The purpose is to fix the function, and let the value of constantly change, so that students can find that is the value of the function corresponding to any given and is the value of the function when, and then know the difference and connection between and.

For another example, when discussing the three elements of function, teachers can set this question: What is the domain, correspondence, and range of the primary function, the inverse proportional function, and the quadratic function respectively? Please use the definition of the function to describe these three functions. This can help students better understand and memorize the three elements of function and function concept symbols, so they can view and apply the concept more widely.

4.4 Draw a Mind Map and Build a Knowledge System

Mind mapping is an effective graphic tool to express divergent thinking, which uses graphics and a network-like structure to build memory links between topic or key words with images, etc. A mind map can help students build a complete conceptual system for newly learned concepts and improve their ability to solve comprehensive problems. Therefore, after understanding and mastering concepts, teachers should lead students to summarize them by drawing a mind map. Finally, students will

obtain a clear structure of concepts with a comprehensive schema emerging in their minds.

To be more specific, when making a summary of the class, the teacher can first lead students to review what they have learned on the concept of function, and then ask them to draw a mind map. Some students can be invited to introduce their versions while the rest can make a comparison so as to refine theirs. The teacher can appraise those good ones and give suggestions on ones that can be improved. Additionally, the teacher's version of the mind map (such as Figure 1) shall be displayed on the screen if the multimedia equipment is available in the classroom. This whole process will enable students to figure out the relationship between and structure of various mathematical knowledge. If time permits, the teacher can play some videos that introduce the evolution of function, which relies heavily on mathematicians' persistent exploration, to encourage and inspire students.

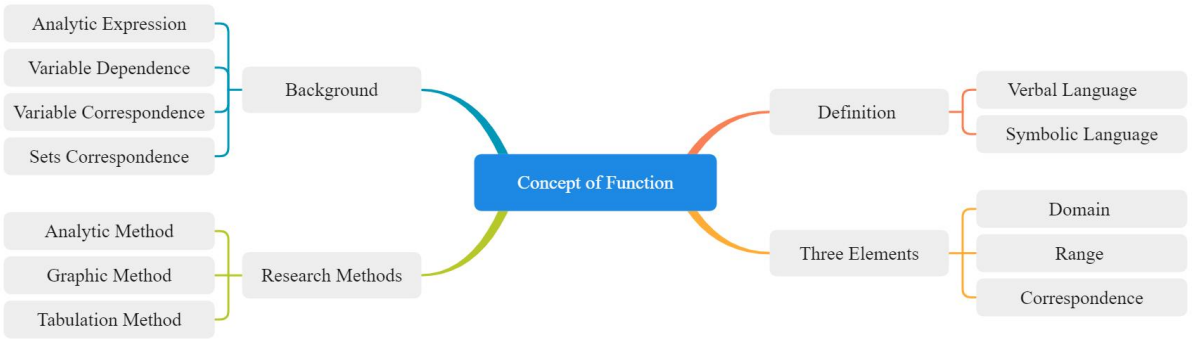

Figure 1.

5. Conclusion

Mathematical concept teaching lays a foundation for teaching all other knowledge. Only by deeply understanding the nature of concepts can students learn them more effectively. There are many concepts that they need to learn in senior high school. This would be a difficult process since concepts are abstract. Despite this, they need to overcome all difficulties. The APOS theory, if applied in mathematical concept teaching in a proper way, will significantly contribute to the teaching content design and summary and teaching evaluation. Mathematical concept teaching in senior high school still faces many problems. In this paper, we analyze the root causes and propose four strategies based on the four stages of the APOS theory to facilitate front-line teachers in teaching mathematical concepts. This theory benefits both teaching and learning in senior high school. Still, mathematics teachers shall gain deep insight into this theory before applying it in practice.

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